DYNAMICS OF MACHINES

5th SEMESTER

Sub code:18ME52

DEPARTMENT OF MECHANICAL ENGG

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MODULE-01

STATIC FORCE ANALYSIS

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Introduction:

Static equilibrium:

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Static force analysis of four bar and single slider mechanism

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Objectives

- To analyze static force analysis of four bar chain mechanism
- To analyze static force analysis of slider crank mechanism

TOM is divided into two parts:-

Kinematics of Machinery: Study of motion of the components and basic geometry of the mechanism and is not concerned with the forces which cause or affect motion. Study includes the determination of velocity and acceleration of the machine members

Dynamics of Machinery: Analyses the forces and couples on the members of the machine due to external forces (static force analysis) also analyses the forces andcouples due to accelerations of machine members(Dynamic force analysis)

Deflections of the machine members are neglected in general by treating machine members as rigidbodies (also called rigid body dynamics). In other words the link must be properly designed to withstand the forces without undue deformation to facilitate proper functioning of the system.

In order to design the parts of a machine or mechanism for strength, it is necessary to determine the forces and torques acting on individual links. Each component however small, should be carefully analysed for its role in transmitting force.

The forces associated with the principal function of the machine are usually known or assumed.

Ex:

a) Piston type of engine: gas force on the piston is known or assumed
b) QRM – Resistance of the cutting tool is assumed

a & b are called static forces.

Example of other static forces are:

i. Energy transmitted
ii. Forces due to assembly
iii. Forces due to applied loads
iv. Forces due to changes in temperature
v. Impact forces

vi. Spring forces
vii. Belt and pulley
viii. Weights of different parts

Apart from static forces, mechanism also experiences inertia forces when subjected to acceleration, called dynamic forces.

Static forces are predominant at lower speeds and dynamic forces are predominant at higher speeds.

Force analysis:

The analysis is aimed at determining the forces transmitted from one point to another, essentially from input point to out put point. This would be the starting point for strength design of a component/ system, basically to decide the dimensions of the components

Force analysis is essential to avoid either overestimation or under estimation of forces on machine member.

Under estimation: leads to design of insufficient strength and to early failure. Overestimation: machine component would have more strength than required. Over design leads to heavier machines, costlier and becomes not competitive

Graphical analysis of machine forces will be used here because of the simplification it offers to a problem, especially in cases of complex machines. Moreover, the graphical analysis of forces is a direct application of the equations of equilibrium.

General Principle of force analysis:

A machine / mechanism is a three dimensional object, with forces acting in three dimensions. For a complete force analysis, all the forces are projected on to three mutually perpendicular planes. Then, for each reference plane, it is necessary that, the vector sum of the applied forces in zero and that, the moment of the forces about any axis perpendicular to the reference plane or about any point in the plane is zero for equilibrium.

That is $\sum F = 0$ & $\sum M = 0$ or

$\sum F x=0$ & $\sum Fy=0$ and $\sum M=0$

A force is a vector quantity and three in properties define a force completely;

Magnitude

Direction

Point of application

Static equilibrium.

Equilibrium

For a rigid body to be in Equilibrium

i) Sum of all the forces must be zero

ii) Sum of all the moments of all the forces about any axis must be zero

i.e, (i) $\sum F = 0$ (ii) $\sum M = 0$

or $\sum Fx = 0$ $\sum TM = 0$

$\sum Fy = 0$ $\sum My = 0$

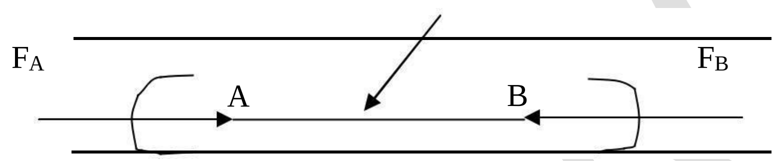
$\sum Fz = 0$ $\sum Tz = 0$

(For a planar system represented by 2D vectors)

Fx, *Fy*, *Fz* force Components along X, Y & Z axis

Equilibrium of two and three force members.

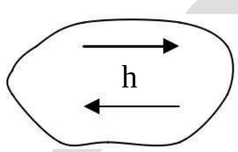
(i) Equilibrium of a body under the action of two forces only (no torque)



For body to the in Equilibrium under the action of 2 forces (only), the two forces must the equal opposite and collinear. The forces must be acting along the line joining A&B.

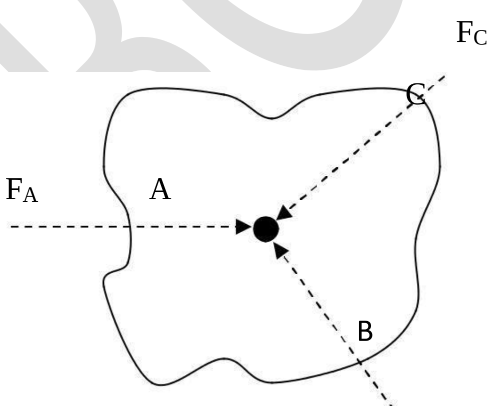
That is,

$F_A = - F_B$ (for equilibrium)



If this body is to be under equilibrium „h" should tend to zero

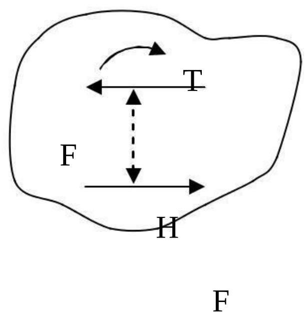
(ii) Equilibrium of a body under the action of three forces only (no torque / couple)



For equilibrium, the 3 forces must be concurrent and the force polygon will be a triangle.

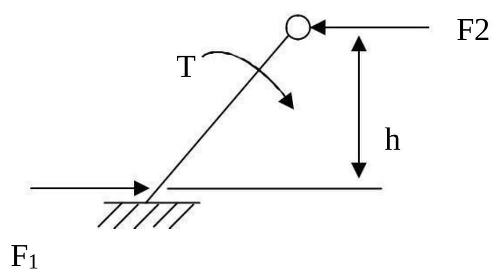
Members with two forces and torque.

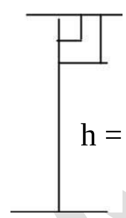
(iii) **Equilibrium of a body acted upon by 2 forces and a torque.**



For equilibrium, the two forces must form a counter couple. Therefore the forces must be equal, opposite and parallel and their senses must be so as to oppose the couple acting on the body

<u>Example:</u>





h = Perpendicular distance between $F1$ & F_2

Free body diagram

The mass is separated from the system and all the forces acting on the mass are represented.

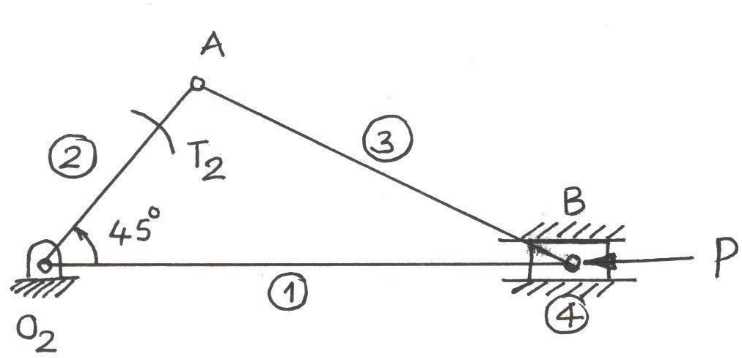
slider-crank mechanism with and without friction.

$P = (8 \times 10^4) \times (0.1)$

$= 8 \times 10^3\ N$

Problem No.1: Slider crank mechanism

Figure shows a slider crank mechanism in which the resultant gas pressure 8 x 10^4 Nm^{-2} acts on the piston of cross sectional area 0.1 m^2. The system is kept in equilibrium as a result of the couple applied to the crank 2, through the shaft at O_2. Determine forces acting on all the links (including the pins) and the couple on 2.

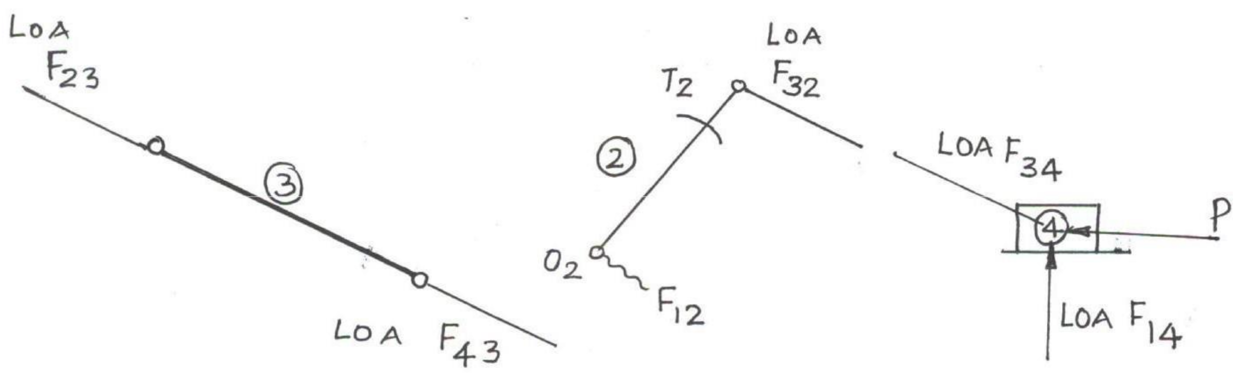


$P = (8 \times 10^4) \times (0.1)$

$= 8 \times 10^3\ N$

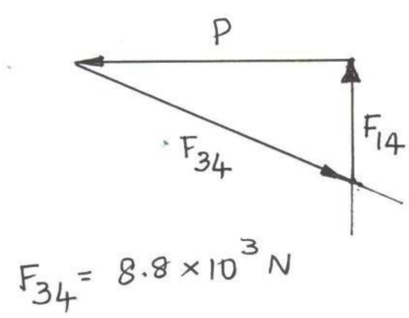
Free body diagrams.

Free body diagram

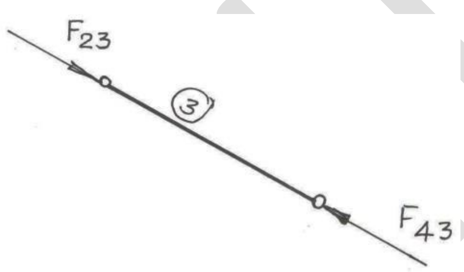


Force triangle for the forces acting on ◯ is drawn to some suitable scale.

Magnitude and direction of P known and lines of action of F_{34} & F_{14} known.



Measure the lengths of vectors and multiply by the scale factor to get the magnitudes of F_{14} & F_{34}. Directions are also fixed.

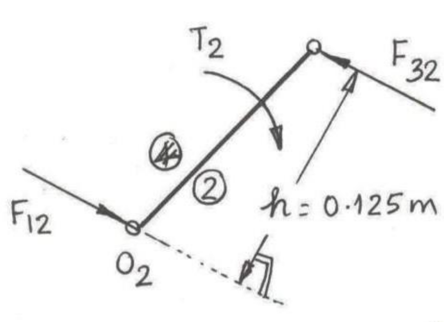


i.e, $F_{23} = -F_{32}$

Since link 3 is acted upon by only two forces, F_{43} and F_{23} are collinear, equal in magnitude and opposite in direction

$$i.e., \quad F_{43} = -F_{23} = 8.8 \times 10^3\ N$$

Also, $F_{23} = -F_{32}$ (equal in magnitude and opposite in direction).



Link 2 is acted upon by 2 forces and a torque (stated in the problem), for equilibrium the two forces must be equal, parallel and opposite and their sense must oppose T_2.

There fore,

$F_{32} = -F_{12} = 8.8 \times 10^3\ N$

F_{32} & F_{12} form a counter clock wise couple of magnitude,

$$F_{23} \times h = F_{12} \times h = 8.8 \times 10^3 \times 0.125 = 1100 Nm.$$

To keep 2 in equilibrium, T2 should act clockwise and magnitude is 1100 Nm. Important to note;

i) h is measured perpendicular to F32 & F12;

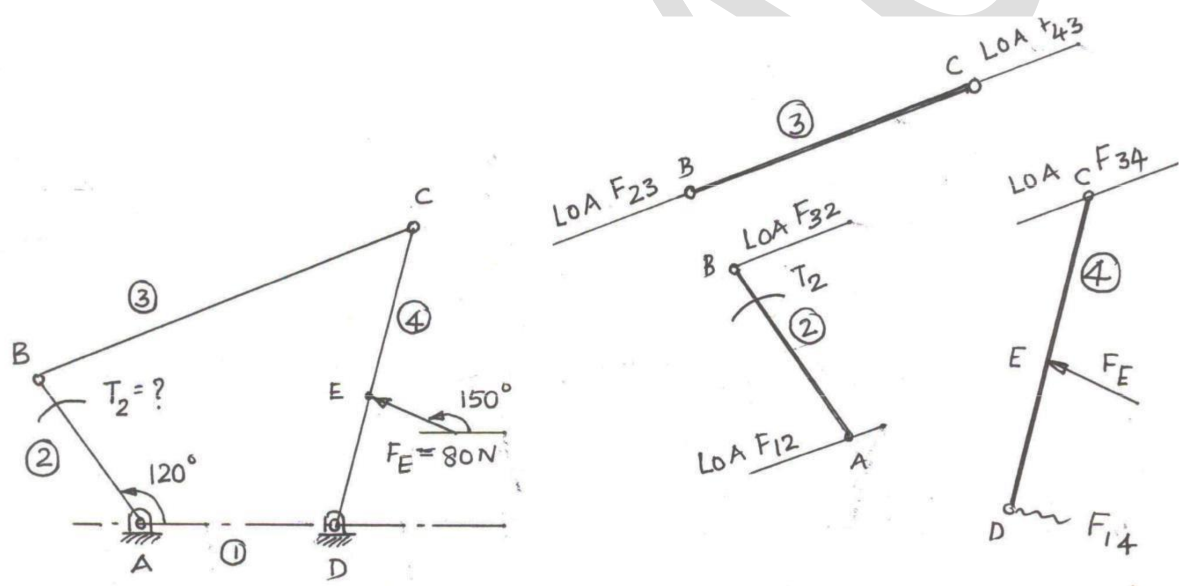
ii) Always multiply back by scale factors.

Static force analysis of four bar and single slider mechanism

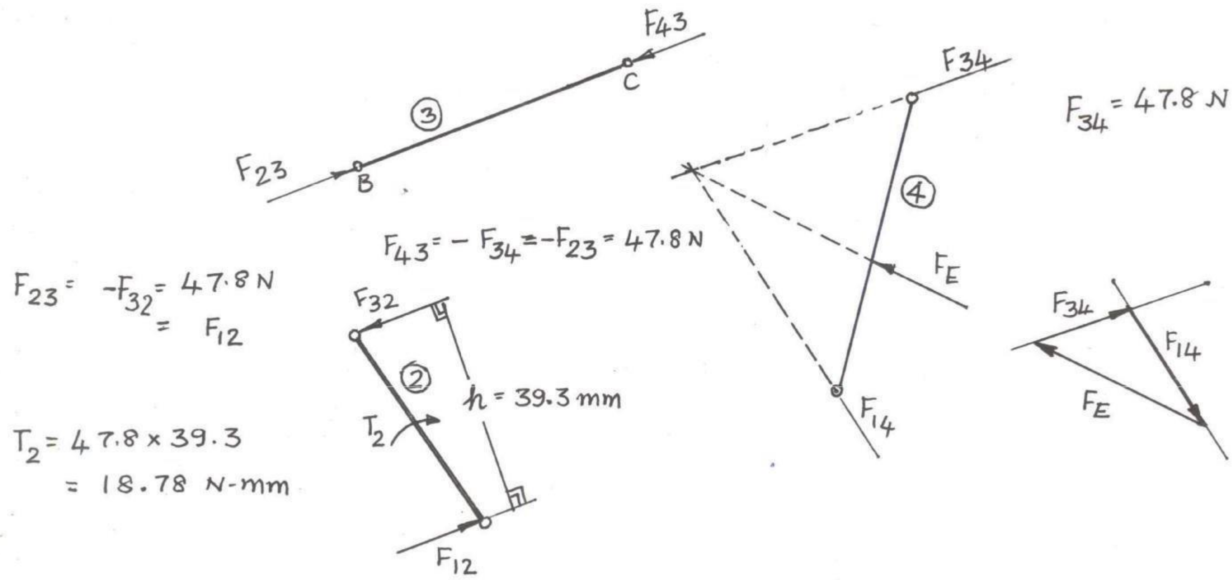
Problem No 2. Four link mechanism.

A four link mechanism is acted upon by forces as shown in the figure. Determine the torque T_2 to be applied on link 2 to keep the mechanism in equilibrium.

AD=50mm, AB=40mm, BC=100mm, Dc=75mm, DE= 35mm,



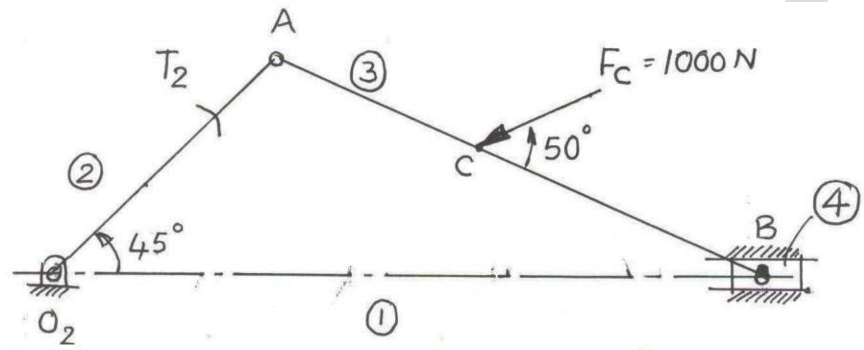
Link 3 is acted upon by only two forces F_{23} & F_{43} and they must be collinear & along BC. Link 4 is acted upon by three forces F_{14}, F_{34} & F_4 and they must be concurrent. LOA F_{34} is known and F_E completely given.

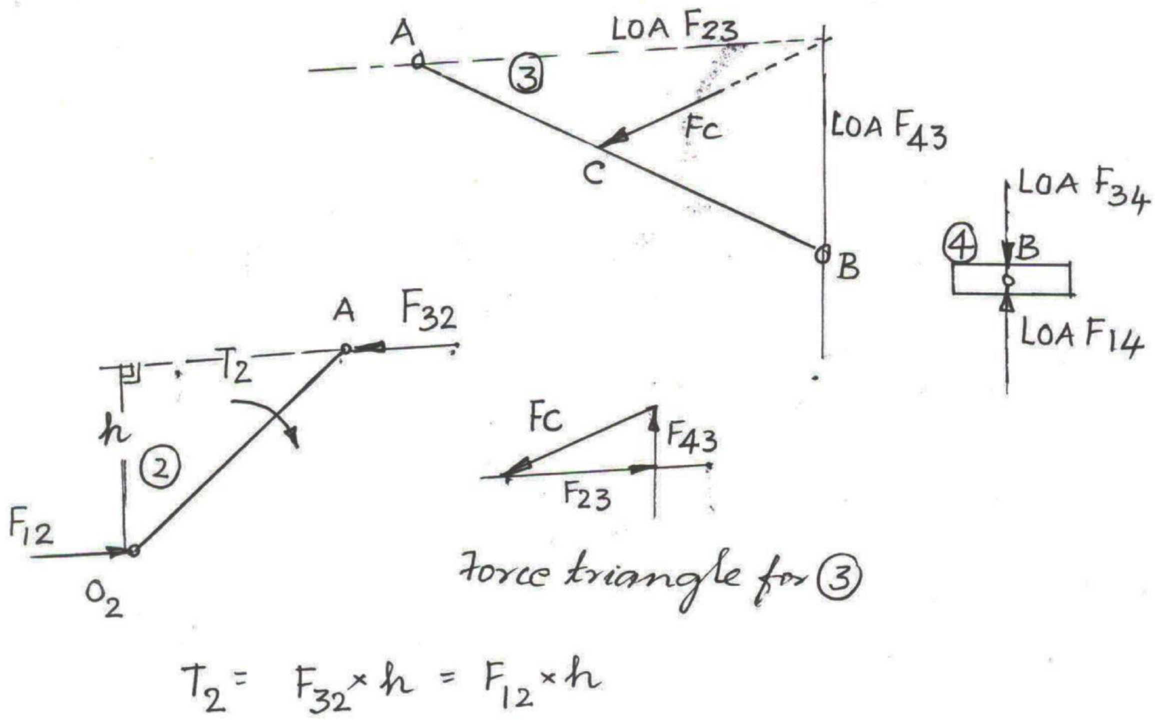


Problem No 3.

Determine T_2 to keep the mechanism in equilibrium

AC=70mm,
AB=150mm,
O_2A= 40mm



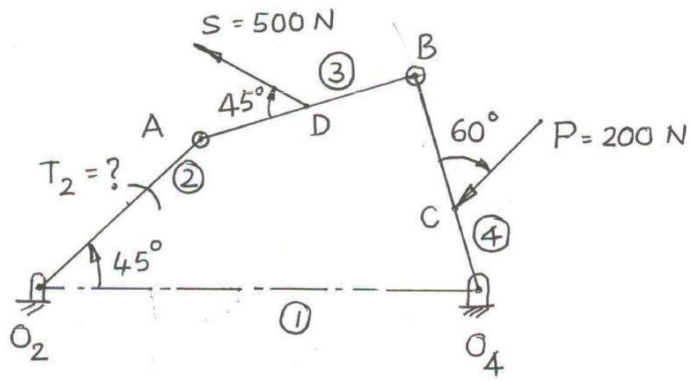


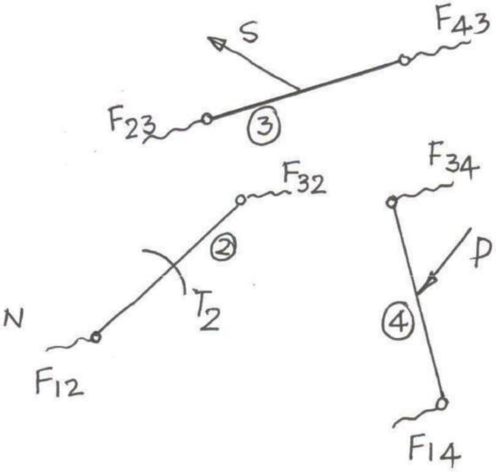
F_{32} and F_{12} form a CCW couple and hence T_2 acts clock wise.

Problem No 4.

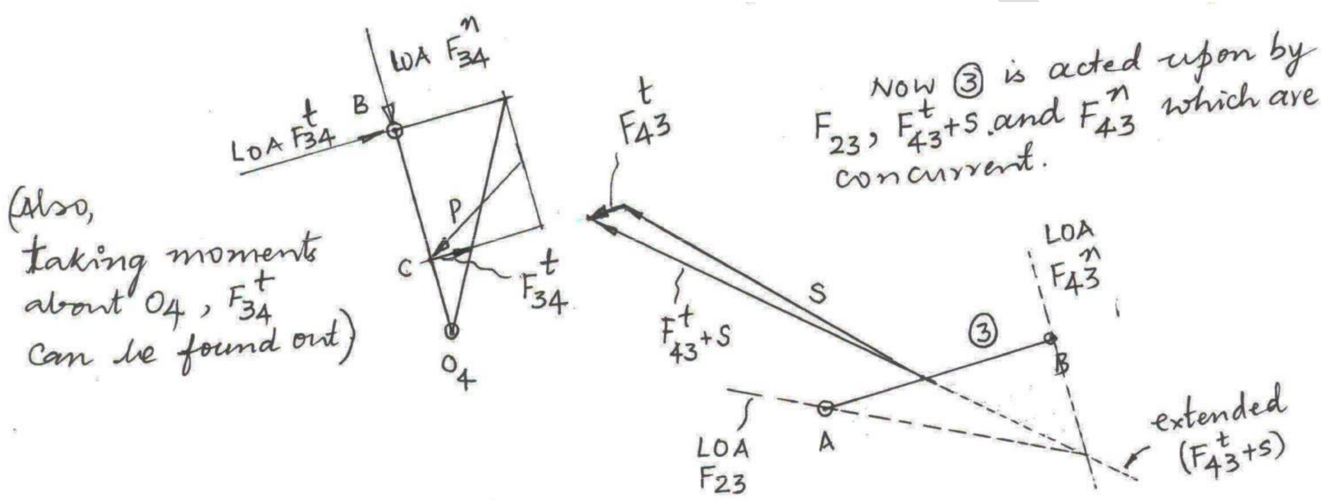
Determine the torque T_2 required to keep the given mechanism in equilibrium.

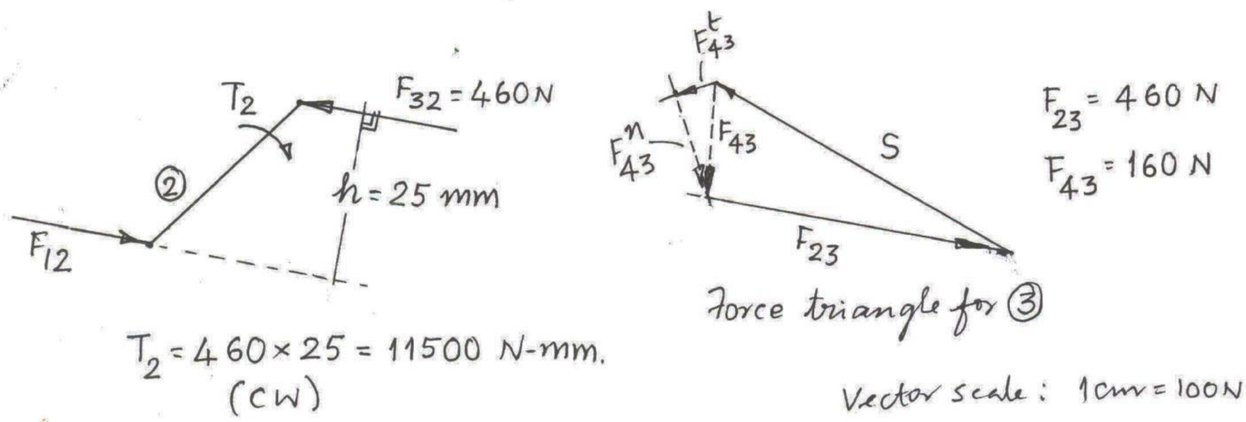
O_2A= 30mm, = AB =O_4B, O_2O_4 =60mm, $A O 2 O_4$ = 60º, BC = 19mm, AD=15mm.





None of the links are acted upon by only 2 forces. Therefore links can"t be analyzed individually.

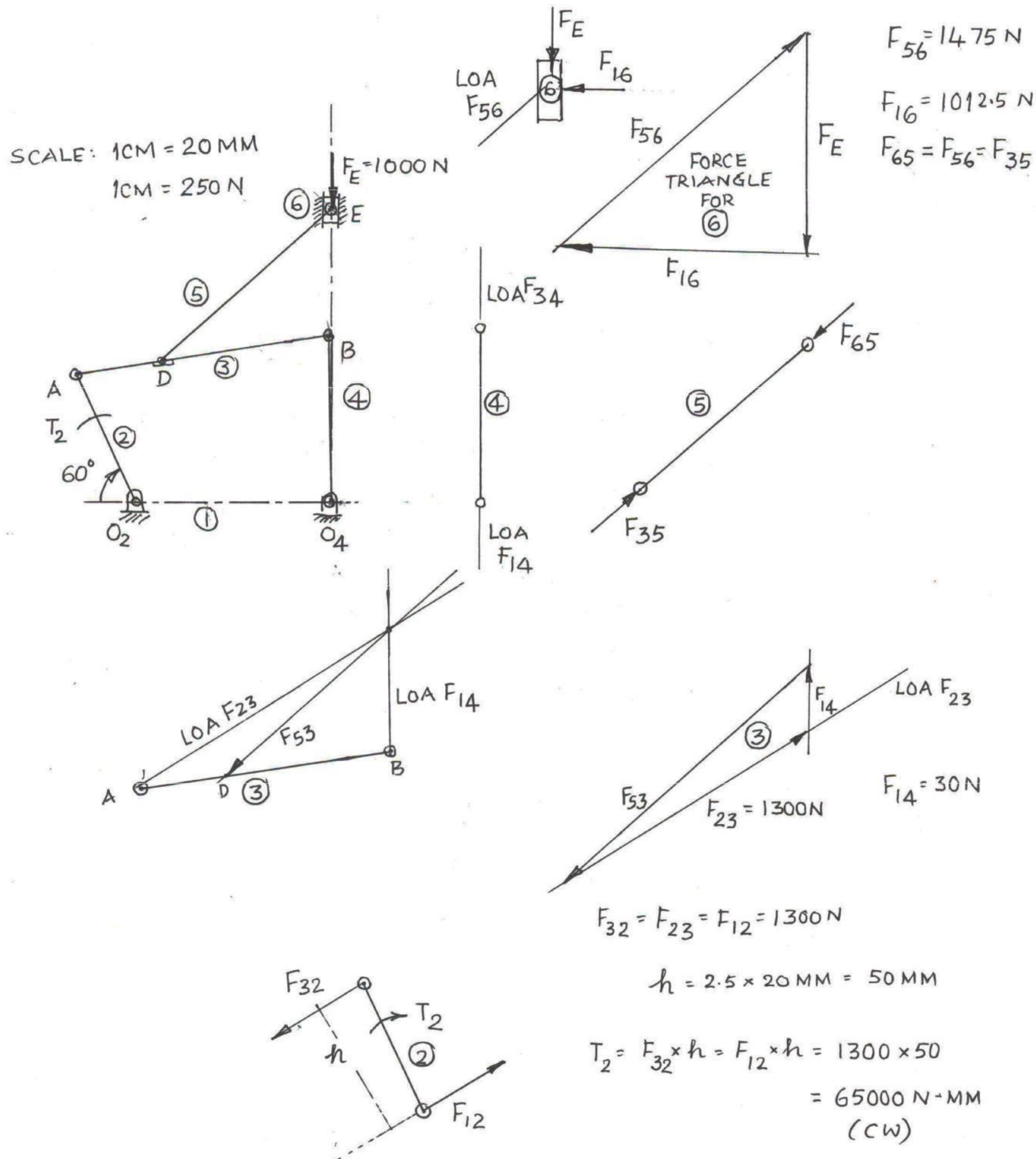




Problem No 5.

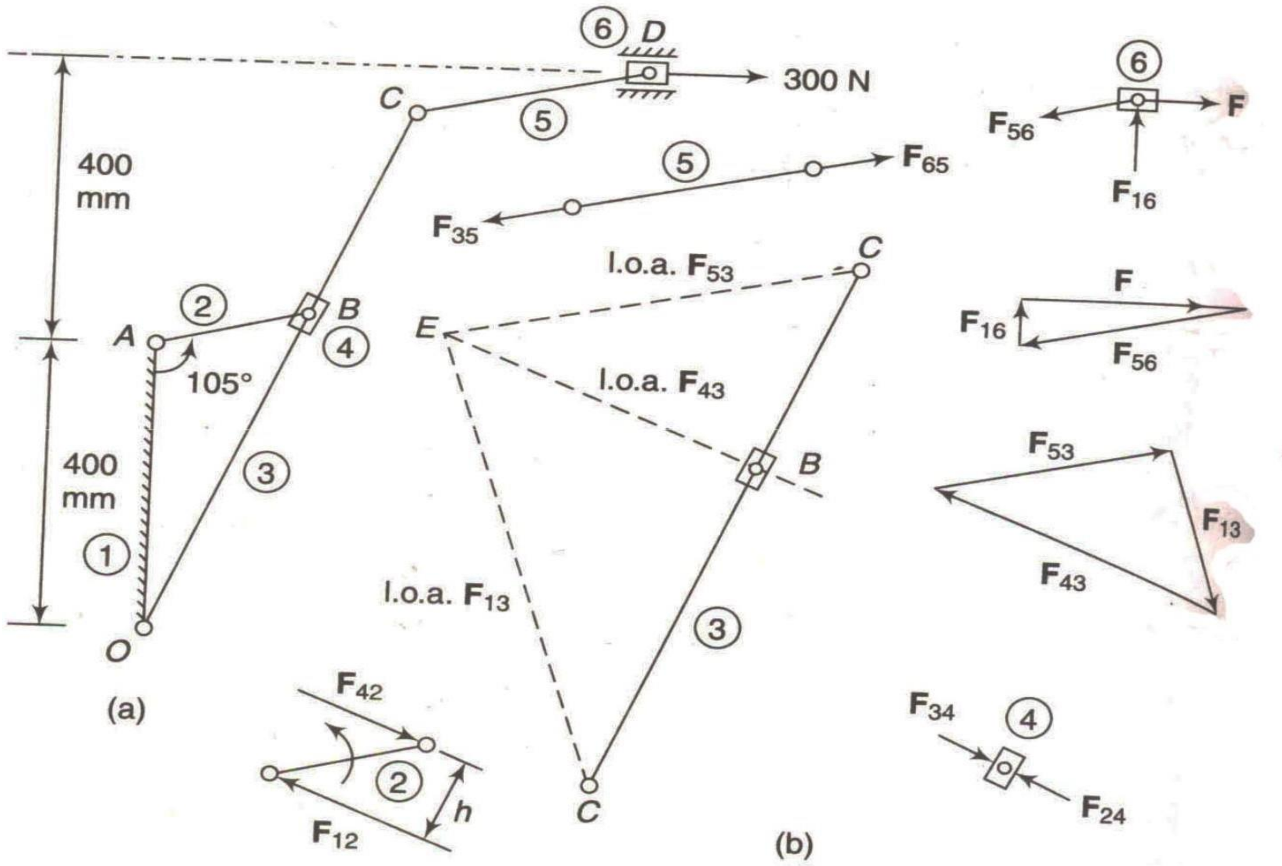
Determine the torque T_2 required to overcome the force F_E along the link 6.

AD=30mm, AB=90mm, O_4 B=60mm, DE=80mm, O_2 A=50mm, O_2 O_4 =70mm



Problem No 6

For the static equilibrium of the quick return mechanism shown in fig. 12.11 (a), determine the input torque T_2 to be applied on link AB for a force of 300N on the slider D. The dimensions of the various links are OA=400mm, AB=200mm, OC=800mm, CD=300mm



Than, torque on link 2,

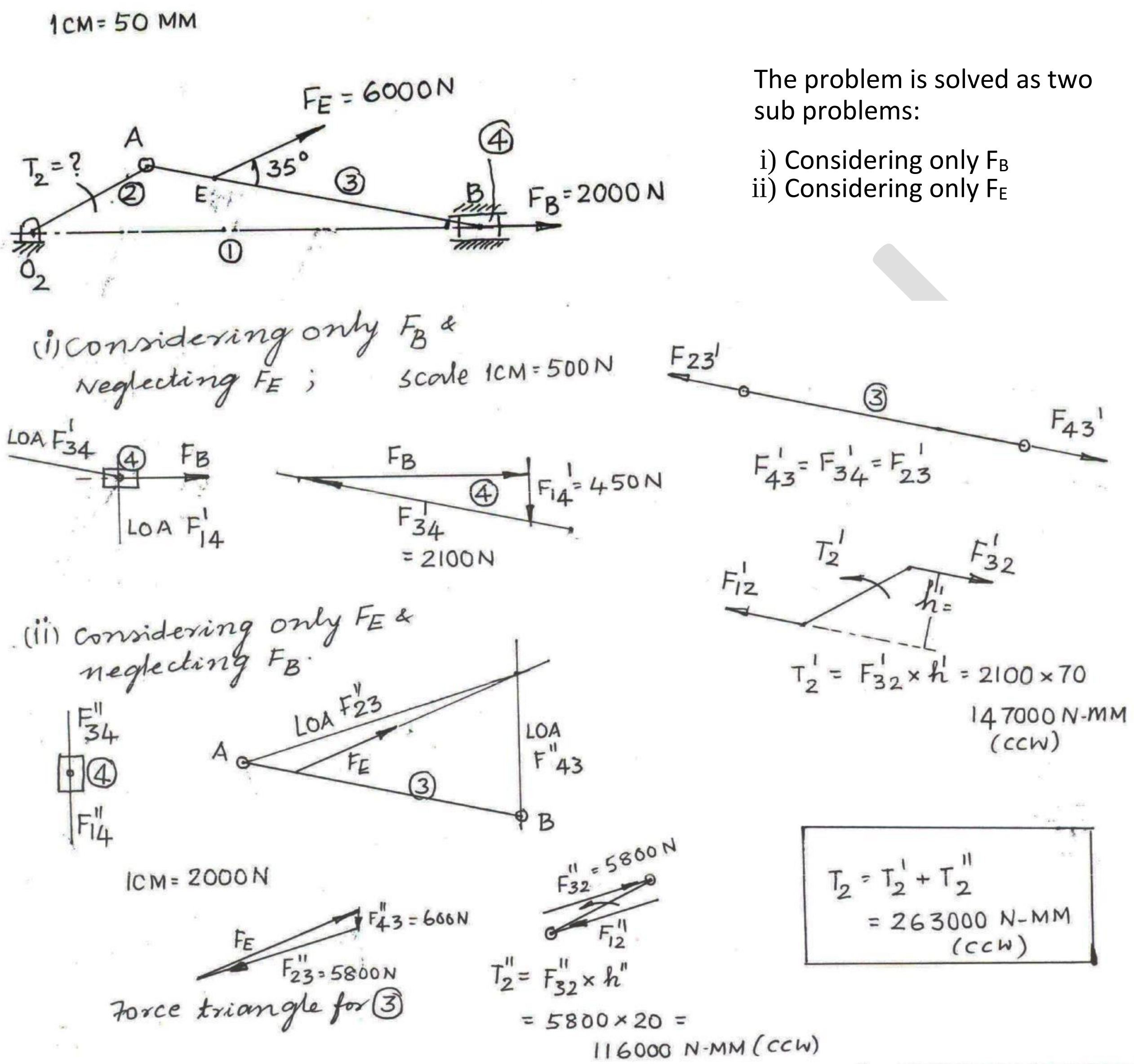
$T_2 = F_{42} x\ h = 403x120 = 48\ 360$ N

Problem No 7.

Determine T_2 to keep the body in equilibrium. O_2A =100MM, AB=250MM,AE=50MM, $AO_2B = 30^0$

The problem is solved as two sub problems:

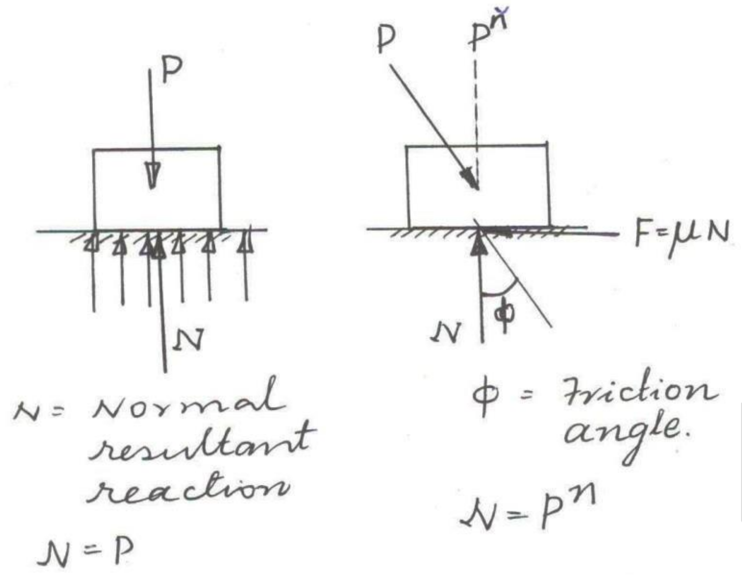
i) Considering only F_B
ii) Considering only F_E



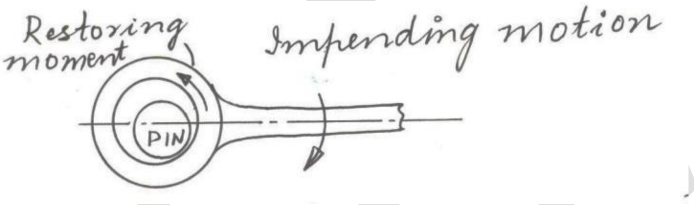
Force Analysis considering friction.

If friction is considered in the analysis, the resultant force on a pin doesn‟t pass through the centre of the pin. Coefficient of friction □ is assumed to the known and is independent of load and speed.

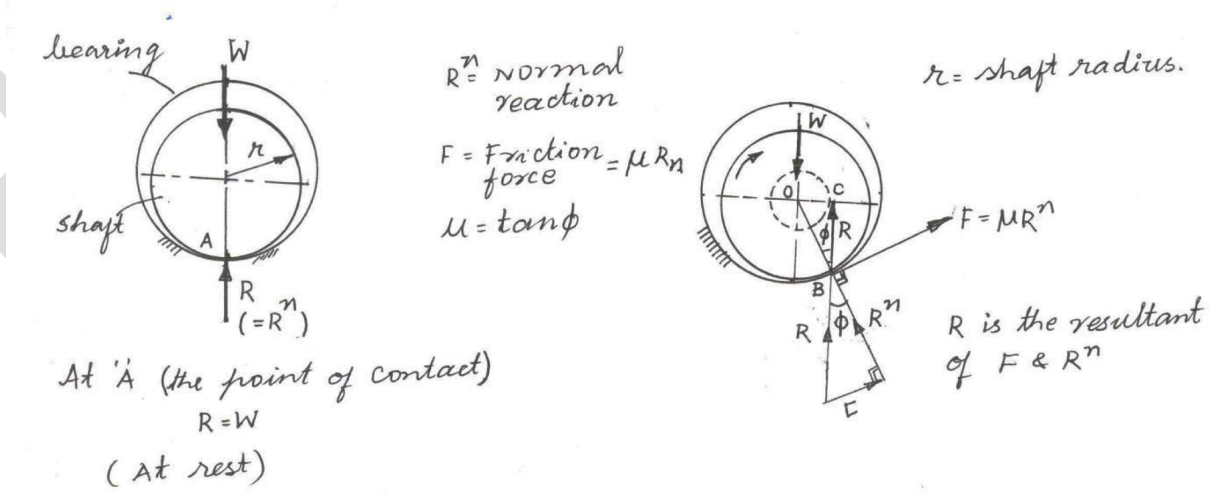
Friction in sliding member.



Friction at pin points (bearings) & friction circle.



When a shaft revolves in a bearing, some power is lost due to friction between surfaces.



While rotating, the point of contact shifts to B; R^n passes through B.The resultant „R" is in a direction opposite to ω.

The circle drawn at O, with OC as radius is called „FRICTION CIRCLE"For the shaft to be in equilibrium; W = R

Frictional moment M = R x OC

W x OC

W x r sin φ

W x r tan φ

(sin φ ≈ tan φ, for small φ) i.e, M = w x r x μ

Radius of the friction circle (OC) = μr.

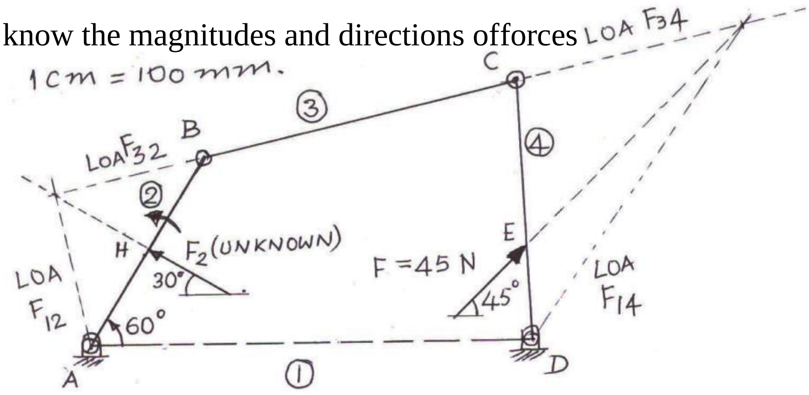
The friction circle is used to locate the line of action of the force between the shaft (pin) and the bearing or a pin joint. The direction of the force is always be tangent to it (friction axis) Friction axis: the new axis along which the thrust acts.

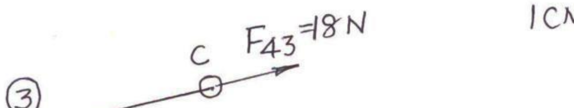
Problem No 8.

In a four bar mechanism ABCD, AB=350mm, BC=50mm, CD=400mm, AD=700mm, DE=150mm, $DAB = 60^0$, AD is fixed. Determine the force on link AB required at the mid point, in the direction shown, for static equilibrium. μ=0.4 for each revolving pair. Assume CCW impending motion of AB. Radius of each journal is 50mm.
Also find the torque on AB for its impending CW motion.

Analysis for CCW motion

Solve the problem neglectingfriction to know the magnitudes and directions offorces





Analysis withFriction considered---AB rotates CCW, DC rotates CCW *ABC* decreasing, LBCD increasing

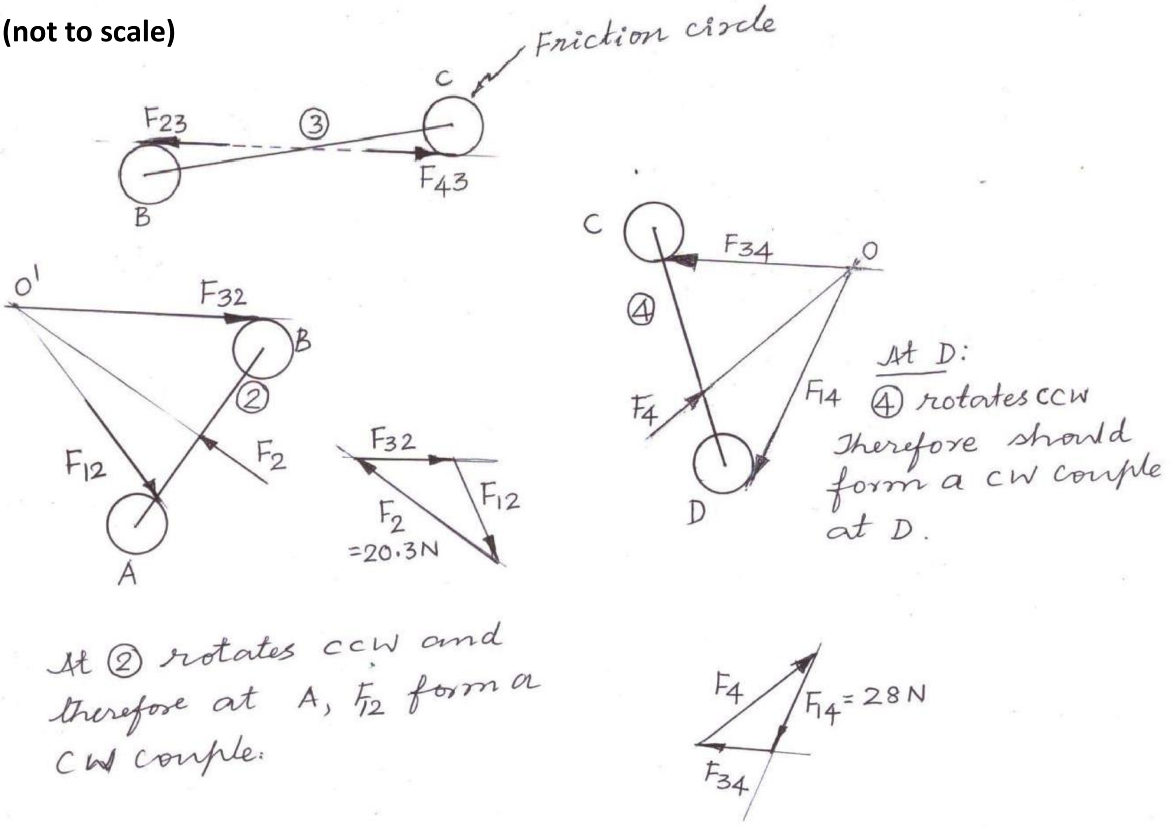
At C:

BCD increases & 3 rotates CW w.r.t 4
Therefore, F_{43} opposes the rotation of 4 by generating a CCW friction couple at C

At B:

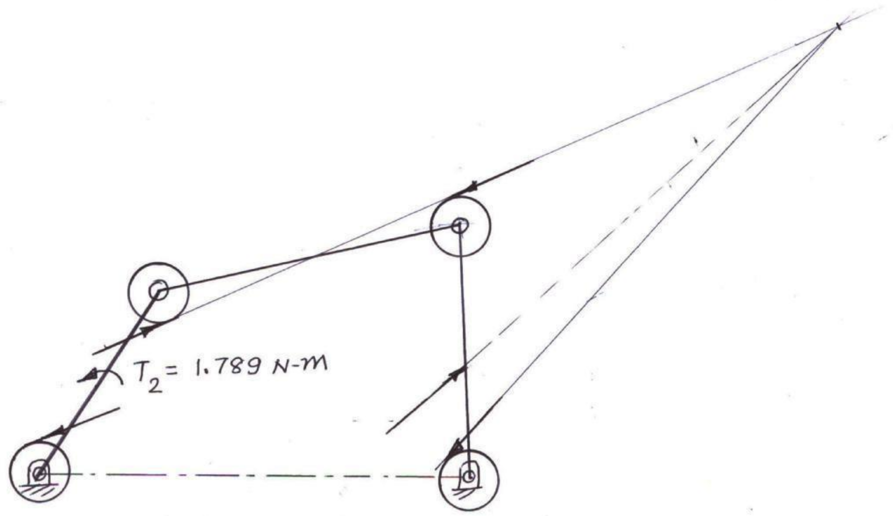
BCD decreases & 3 rotates CW w.r.t 2
Therefore, F_{23} forms a CCW friction couple at B

(not to scale)



For CW rotation of AB

Radius of the friction circle = μ x journal radius = 0.4 x
50 = 20mm



MODULE-02

FLYWHEEL

In practice, there are two following types of cases where reciprocating enginemechanism is used :

(a) An internal combustion engine or a steam engine which is used as a prime mover to drive generators, centrifugal pumps, etc.

(b) A punching machine which is driven by a prime mover like electric motor.

In both these cases either a variable torque is supplied where demand is a constant torqueor demand is variable torque whereas constant torque is supplied. In both these cases there is mismatch between the supply and demand. This results in speed variation. In case of generators, speed variation results in change in frequency and variation in voltage. On the other hand, punching machine requires energy at small interval only when punching is done. To supply such large energy at the time of punching, motor of high power shall be required. At the same time, there will be large variation in speed. To smoothen these variations in torque, flywheel is used which works as a energy storage.
This results in usage of low power motor in punching machine.

Objectives

After studying this unit, you should be able to

- explain the method of drawing turning moment diagram for a prime mover,
- determine the fluctuation of energy in a cycle,
- determine the power of prime power, and
- determine mass moment of inertia of a flywheel and design it.

Turning Moment Diagram:

Figure 4.3 shows a layout of a horizontal engine.

Let p = effective gas pressure on the piston in N/m^2,

A = area of the piston in m^2,

m_{rec} = mass of reciprocating parts, i.e. mass of the piston gudgeon pin and part ofmass of connecting rod 'm_1',

Q = thrust force on the connecting rod in N,

ω = angular velocity of the crank, and

M = Turning moment on the crank.

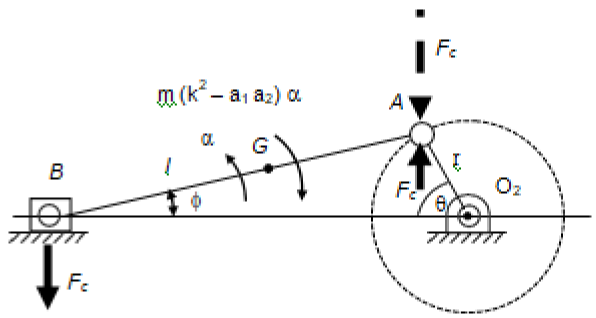


Figure 4.2 : Slider Crank Mechanism

Therefore, $F_C\, l \cos\phi = m\,(k^2 - a_1\, a_2)\,\alpha$

or, $F_C = \dfrac{m\,(k^2 - a_1\, a_2)\,\alpha}{l \cos\phi}$ . . . .

The correction in the turning moment will be equal to the moment of force F_C shown by the dashed line about the crank centre.

$$\text{Correction couple} = -F_C\, r \cos\theta = -\frac{m\,(k^2 - a_1\, a_2)\,\alpha}{l \cos\phi}\, r \cos\theta$$

Let $\dfrac{r}{l} = \lambda$

But $\cos\phi = \left(1 - \dfrac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}$

$$= (1 - \lambda^2 \sin^2\theta)^{\frac{1}{2}} = 1 - \frac{\lambda^2}{2}\sin^2\theta - \frac{\lambda^4}{8}\sin^4\theta + \ldots$$

Differentiating it w.r.t. 't' and dividing it by $\sin\phi = \lambda \sin\theta$

$$\frac{d\phi}{dt} = \omega\left(\lambda \cos\theta + \frac{\lambda^3}{2}\sin^2\theta \cos\theta + \ldots\right)$$

Differentiating again w.r.t. 't' and assuming 'ω' constant

$$\frac{d^2\phi}{dt^2} = \lambda\,(-\omega^2\ (\sin\theta))$$

(By approximation neglecting higher terms)

or, $\dfrac{d^2\phi}{dt^2} = -\omega^2\,\dfrac{l}{r}\sin\theta = \alpha$

Substituting for α

$$\text{Correction couple } M_c = -\frac{m\,(k^2 - a_1\, a_2)\,\omega^2 r^2}{l \cos\phi}\,\frac{\sin 2\theta}{2l} \quad \ldots (11.5)$$

Turning Moment Diagram of a Single Cylinder4-stroke IC Engine

If the effect of correction couple is ignored, the approximate turning moment

$$M = (\text{Gas force} + \text{Inertia force})\ O_2 D$$

The diagram which is plotted for '*M*' against crank angle 'θ' is called turning momentdiagram. This diagram can be plotted progressively as explained below :

(a) There are two forces, i.e. gas force and inertia force.

$$\text{Gas force} = p \times \text{Piston area}$$

where *p* is the gas pressure.

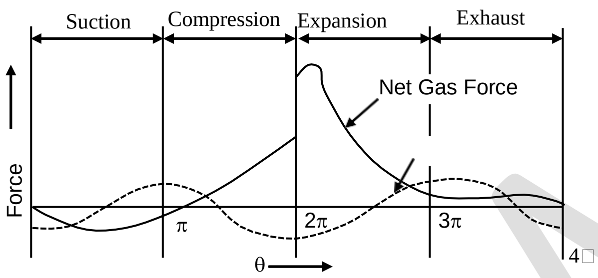


Figure 4.4(a)

The variation in the gas force will be due to the change in pressure. The gasforce and inertia force have been plotted in Figure 4.4(a) for all the four strokes.

(b) The net force is the resultant of gas force and inertia force. It can be plotted in reference to θ as shown in Figure 4.4(b).

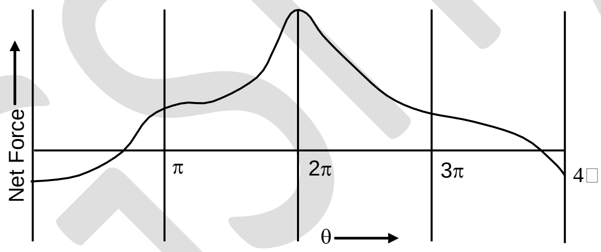
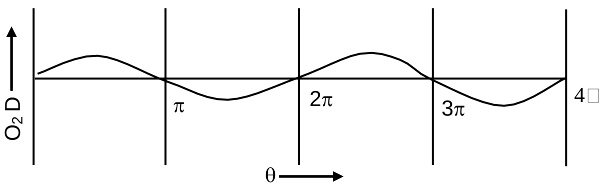


Figure 4.4(b)

(c) The value of O_2D is given by

$$O_2D = r\,(\sin\theta + \cos\theta\,\tan\phi)$$

For various values of θ, O_2D can be determined and then plotted. The plotof this is shown in Figure 4.4(c).



(d) The approximate turning moment ‘M’ = Net force □ O_2D. The plot of ‘M’ Vs □ is shown in Figure 4.4(d).

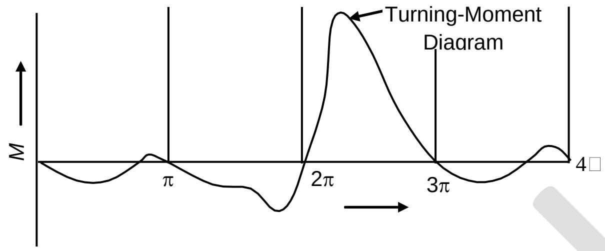


Figure 4.4(d)

The turning moment in the suction stroke and exhaust stroke is very small. In case of compression stroke and expansion stroke turning moment is higher. In compression stroke, energy is to be supplied and in expansion stroke, large amount of energy is available. By surveying the turning moment diagram, it is observed that the energy is supplied in three strokes and energy is available only in one stroke. Therefore, three strokes, i.e. suction stroke, compression, and exhaust stroke the engine is starving of energy and in expansion stroke it is harvesting energy. At the same time it is observedthat there is large variation of turning moment during the cycle. The variation in the turning moment results in corresponding variation in speed of the crank.

Turning Moment Diagram of a Multicylinder 4-stroke IC Engine

In case of multi cylinder engine there will be more expansion strokes. For example, in the case of three cylinder engine, there will be three expansion strokes in each cycle. Incase of 4 cylinder 4-strokes engine there will be four expansion strokes. Therefore, in multi cylinder engine there will be lesser variation in turning moment as compared to single cylinder engine and consequently there is expected to be less variation in speed. The turning moment diagram for a multi cylinder engine is expected to be as shown in Figure 4.5. Therefore the variation in the turning moment reduces with the increase in the number of cylinders.

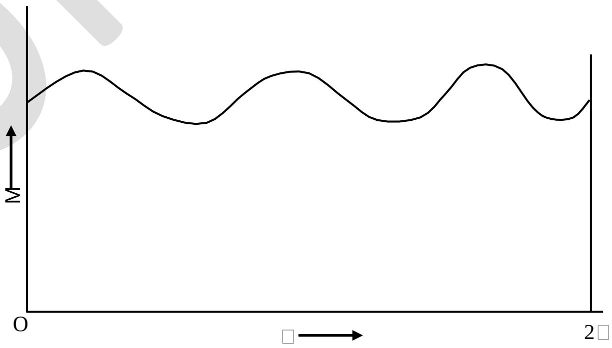
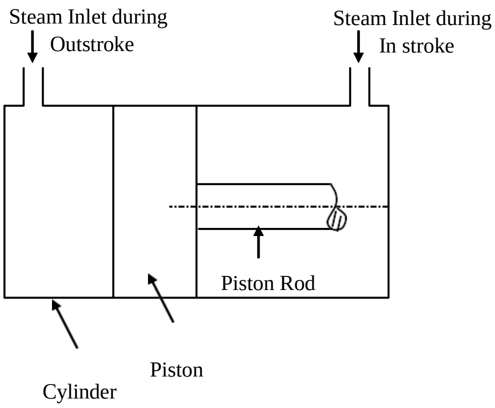


Figure 4.5 : Turning Moment Diagram

Turning Moment Diagram of a Single Cylinder Double Acting Steam Engine

The cylinder and piston arrangement of the steam engine is shown in Figure 4.6(a) and

Flywheel


Steam Inlet during
Outstroke
Steam Inlet during
In stroke
Piston Rod
Cylinder
Piston

(a)

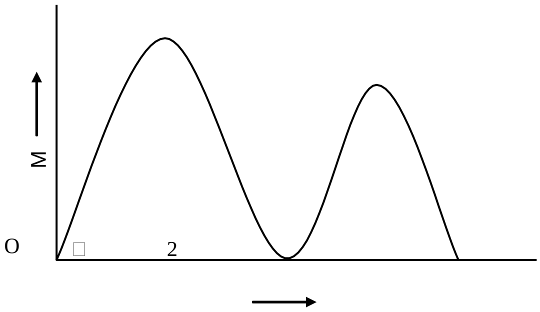

M
O
2

Figure 4.6 : Turning Moment Diagram of a Single Cylinder Double Acting Steam Engine

For outstroke, force = steam pressure □ area of the piston.

For instroke, force = steam pressure □ (area of piston – area of piston rod).

During out stroke the area over which steam pressure acts is more as compared to in stroke where some of the area is occupied by the piston rod. Because of the difference in the available areas there is difference in the maximum turning moments in the two strokes. Steam pressure is nearly constant and variation in the turning moment is due to the value of O_2D and inertia force of the reciprocating masses. As compared to the singlecylinder 4-stroke engine, the variation in turning moment is less in case of double acting steam engine.

As shown in Figures 4.4 to 4.6, the turning moment '*M*' varies considerably whereas theresisting moment say 'M_R' which is due to the machine to be driven remains constant over a cycle for most of the cases. If we superimpose the resisting moment over the turning moment diagram, a situation shown in Figure 4.7 will arise. If M_R is equal to the average turning moment (M_{av}), energy available shall be equal to the energy required over a cycle. It can be observed that for some values of □ turning moment is more than M_R and for some values of □ turning moment is less than M_R.

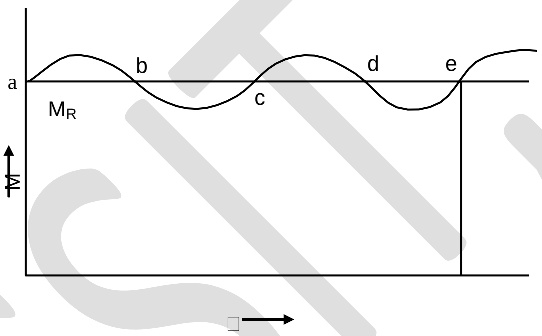


Figure 4.7 : Fluctuation of Energy and Speed

The energy output can be expressed mathematically as follows :

Theory of Machines

The average turning moment for the cycle is

$$M_{av} \square \frac{E}{\text{Angle for cycle}}$$

The angle for the cycle is 2π for the two stroke engines and 4π for four strokes enginesand in case of steam engines it is 2π.

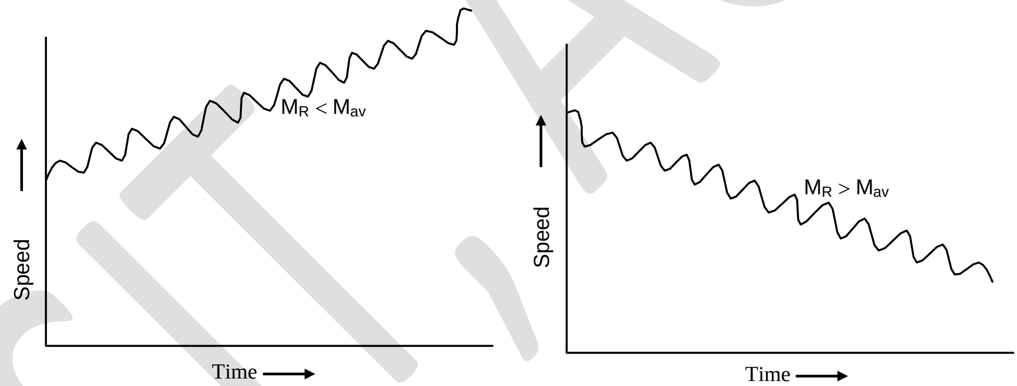
For a stable operation of the system

$$M_R = M_{av}$$

In the stable system, the mean speed remains constant but variation of speed will be there within the cycle. The speed remains same at the beginning and at the end of thecycle.

If $M_R < M_{av}$, the speed increases from cycle to cycle. The speed graph is shown inFigure 4.8(a).

If $M_R > M_{av}$, the speed decreases from the cycle to the cycle. The speed graph is shown inFigure 4.8(b).



(a) **(b)**

Figure 4.8 : Speed Graph

From Figure 4.7, we observe that $M_R = M_{av}$ at points *a*, *b*, *c*, *d* and *e*. Since $M > M_R$ from*a* to *b*, speed of the crank shaft will increase during this period. From *b* to *c* $M < M_R$ andspeed will decrease. Similar situation will occur for *c* to *d* and *d* to *e*. At *e* the cycle is complete and the speed at *e* is same as that of *a*. The energy at all these points can be determined.

□*b* □*d*

$E_b \square E_a \square$

□

□*a*

□*c*

$E_c \square E_b \square$

□

□*b*

$(M - M_R)\, d\theta$

$$E_d - E_c = \int_{\theta_c}^{\theta_e} (M - M_R)\, d\theta \qquad (M - M_R)\, d\theta$$

$E_e - E$

Flywheel design:

It has been discussed in the preceding section that fluctuation of energy results in fluctuation of the crank shaft speed which then results in fluctuation of the kinetic energy of the rotating parts. But the maximum permissible fluctuation in speed of the crank shaft is determined by the purpose for which the engine is to be used. Therefore, tokeep the maximum fluctuation of speed within a specific limit for a given maximum fluctuation of energy, a flywheel is mounted on the crank shaft.

Mass Moment of Inertia of Flywheel for an IC Engine

The function of the flywheel is to store excess energy during period of harvestation andit supplies energy during period of starvation. Thereby, it reduces fluctuation in the speed within the cycle. Let ω_1 be the maximum angular speed and ω_2 be the minimum angular speed.

Let *I* be the mass moment of inertia of the flywheel.

Neglecting mass moment of inertia of the other rotating parts which is negligible incomparison to mass moment of inertia of the flywheel.

Maximum kinetic energy of flywheel

$$(\text{K.E.})_{max} = \frac{1}{2} I \omega_1^2$$

Minimum kinetic energy of flywheel

$$(\text{K.E.})_{min} = \frac{1}{2} I \omega_2^2$$

Change in K.E., i.e. $\Delta \text{K.E.} = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$

Δ K.E. = fluctuation in energy, i.e. ΔE

$$\therefore \quad \Delta E = \frac{1}{2} I (\omega_1^2 - \omega_2^2) \qquad \text{. . . (4.12)}$$

or, $$\Delta E = \frac{1}{2} I (\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= \frac{1}{2} I (\omega_1 - \omega_2) \times 2\omega$$

where 'ω' is average speed given by

$$\omega = \frac{(\omega_1 + \omega_2)}{2}$$

or, $$\Delta E = \frac{1}{2} I \frac{(\omega_1 - \omega_2)}{w} \times \omega^2$$

or, $$\Delta E = I\, k_s\, \omega^2$$

Energy fluctuation can be determined from the turning
value of k_s, and given value of speed ω, I can be deterı
Eq. (4.12) can also be written as follows :

$$\Delta E = \frac{1}{2} M\, k^2\, (\omega_1^2 - \omega_2^2)$$

where k is the radius of gyration and M is mass of the

or, $$\Delta E = \frac{1}{2} M\, \{(k\omega_1)^2 - (k\omega_2)^2\}$$

Let V_1 be the maximum tangential velocity at the radiu
minimum tangential velocity at the radius of gyration

∴ $$V_1 = k\omega_1 \quad \text{and} \quad V_2 = k\omega_2$$

and $$\Delta E = \frac{1}{2} M\, (V_1^2 - V_2^2)$$

It can be observed from Eq. (4.13) that

(a) The flywheel will be heavy and of large s
is limited by the practical considerations.
engine shall require larger flywheel as co
engine.

(b) For slow speed engine also the flywheel ı
high value of I required.

(c) For high speed engines, the size of flywh
because of lower value of I required.

(d) If system can tolerate considerably highe
flywheel will also be smaller for same va

Example 4.1

The turning moment diagram for a multi cylinder IC engine is drawn to thefollowing scales

1 cm =

15º crank

angle1 cm

= 3 k Nm

During one revolution of the crank the areas with reference to the mean torque lineare 3.52, (□) 3.77, 3.62, (□) 4.35, 4.40 and (–) 3.42 cm^2. Determine mass moment of inertia to keep the fluctuation of mean speed within □ 2.5% with reference to mean speed. Engine speed is 200 rpm.

Solution:

The turning moment diagram is shown in Figure 4.9.The scales are

1 cm =

15º crank

$$\text{Therefore,} \quad 1\,\text{cm}^2 = 3000 \times \frac{\pi}{180} \times 15 = 785\ \text{Nm}$$

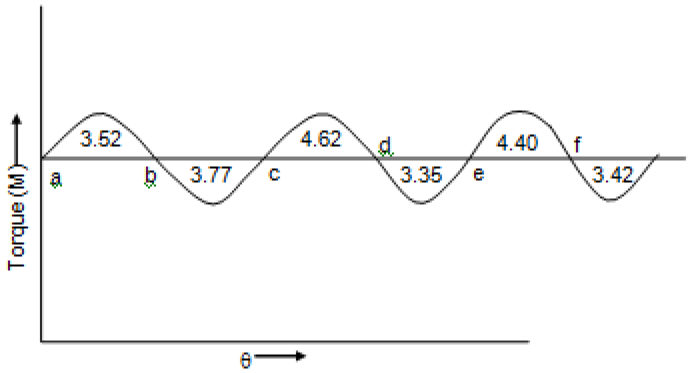


Figure 4.9 : Figure for Example 4.1

The overall speed fluctuation = 2 × 2.5%

∴ Coefficient of speed fluctuation 'k_s' = 0.05

Engine speed = 200 rpm

∴ $\omega = \frac{2\pi \times 200}{60} = 20.93$ r/s

Let Energy level at a is 'E' cm²

Energy level at b is $E_b = E + 3.52$

Energy level at c is $E_c = E_b - 3.77 = E + 3.52 - 3.77 = E - 0.25$

Energy level at d is $E_d = E_c + 3.62 = E - 0.25 + 3.62 = E + 3.37$

Energy level at e is $E_e = E_d - 4.35 = E + 3.37 - 4.35 = E - 0.98$

Energy level at f is $E_f = E_e + 4.40 = E - 0.98 + 4.40 = E + 3.42$

Energy level at g is $E_g = E_f - 3.42 = E + 3.42 - 3.42 = E$

Energy level at the end of cycle and at the beginning of the cycle should be same.
By comparing the values of energies at various points, we get

Maximum energy is at 'b', i.e. $E_{max} = E + 3.52$

Minimum energy is at 'e', i.e. $E_{min} = E - 0.98$

Since, $\Delta E = I\ k_s\ \omega^2$

∴ $3532.5 = I \times 0.05 \times (20.93)^2$

or, $I = \frac{3532.5}{0.05 \times (20.93)^2} = \frac{3532.5}{20.9} = 169$ kgm²

Mass Moment of Inertia of Flywheel for a Punching Press

In this case torque supplied is constant because these machines are driven by the electric motor but the demand torque, i.e. resisting torque varies during cycle. The example of them are punching press, shearing machine, etc.

The schematic diagram of punching press is shown in Figure 4.13. In place of slider in slider crank mechanism, punching tool is used. Since motor is used to drive this press, the torque supplied shall be constant. On the other hand, high resisting torque will act when punching operation is done, i.e. from $\theta = \theta_1$ to θ_2. After this operation the resisting torque will be almost zero. Unless a flywheel is used, the speed of the crank shaft will be very high when resisting torque is very small and substantial decrease in speed shall take place when punching operation is done. If flywheel is provided, the excess energy shall be absorbed in the flywheel and it will be available when punching operation is being done where energy is deficient. It will result in reduction of the power of motor required if a suitable flywheel is used.

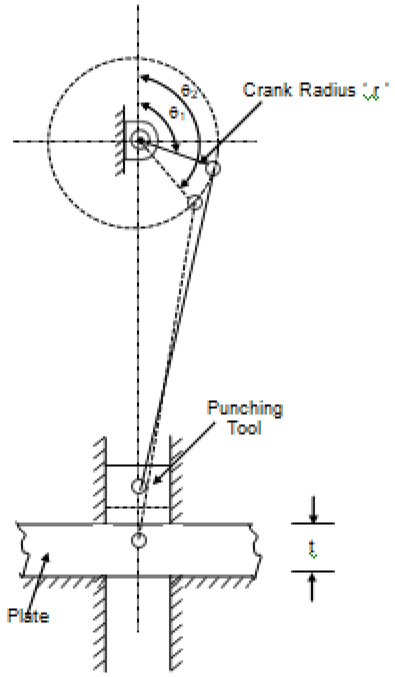


Figure 4.13 : Punching Press

Let E be the energy required for punching one hole. For a stable operation, the energy supplied to the crank for one revolution should also be equal to E.

The fluctuation of energy 'ΔE' = Energy required for one punch – Energy supplied during punching

Energy E = Work done in punching hole

$$= \left(\frac{1}{2}\right)\{\text{Maximum force } (P)\} \times \text{Thickness of plate}$$

Here, P = Sheared area × Shear strength

Angles θ_1 and θ_2 should be in radians.

Let t be the thickness of the plate in which holes are to be punched.

s be the length of the stroke.

r be the length of the crank.

l be the length of the connecting rod.

θ_1 and θ_2 can be determined geometrically if r, l and t are known.

The rough estimate can also be made as follows :

$$\frac{\theta_2 - \theta_1}{2\pi} \approx \frac{t}{2s} = \frac{t}{4r}$$

$$\left(\frac{(\theta_2 - \theta_1)}{} \right) = E\left(1 \quad \frac{t}{} \right) = I\ k\ \omega^2$$

The mass moment of inertia of the flywheel for a given value of coefficient of fluctuation of speed can be determined. In order to reduce the size of flywheel it will be better to mount the flywheel on a shaft having higher value of 'ω'.

Example 4.6

A punching machine punches 6 holes per minute. The diameter of each hole is 4 cm and thickness of the plate is 3 cm. The stroke of the punch is 10 cm. The work done per square cm of sheared area is 600 J. The maximum speed of the flywheel at its radius of gyration is 28 m/s. Determine mass of the flywheel required so that its speed at its radius gyration does not fall below 26 m/s. Determine power of the motor required.

Solution

The sheared area of the hole = Circumference × depth

$$= \pi \times 4 \times 3 = 12\pi = 37.68 \text{ cm}^2$$

Energy required per hole $E = 37.68 \times 600 = 22608$ J

Number of holes per minute = 6

$\therefore$ Energy required per minute = 6×22608

$\therefore$ Energy required per second = $\dfrac{6 \times 22608}{60} = 2260.8$

$\therefore$ Power of motor required = $\dfrac{2260.8}{1000} = 2.2608 \text{ kW} = 2.26 \text{ kW}$

Fluctuation of energy $\Delta E = I k_s \omega^2 = \dfrac{1}{2} I (\omega_1^2 - \omega_2^2)$

$$= \frac{M}{2} k^2 (\omega_1^2 - \omega_2^2)$$

$$= \frac{M}{2} \{(k\omega_1)^2 - (k\omega_2)^2\}$$

$$= \frac{M}{2} (V_{max}^2 - V_{min}^2)$$

The fluctuation of energy

$$= \left(\frac{\text{Thinckess of plate}}{2 \times \text{stroke}} \right)$$

$$\left(\frac{3}{\quad} \right) \quad \left(\frac{3}{\quad} \right)$$

$$= 22608\,(1 - 0.15)$$

$$= 22608 \times 0.85 = 19216.8 \text{ J}$$

$\therefore$ $19216.8 = \dfrac{M}{2}(28^2 - 26^2) = \dfrac{108\,M}{2} = 54\,M$

or, $M = 355.87$ kg.

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MODULE-03

BALANCING OF ROTATING MASSES

Introduction

The high speed of engines and other machines is a common phenomenon now-a-days. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses Balancing of Rotating Masses We have already discussed, that whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass. This is done in such a way that the centrifugal force of both the masses is made to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called balancing of rotating masses.
The following cases are important from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.

2. Balancing of a single rotating mass by two masses rotating in different planes.

3. Balancing of different masses rotating in the same plane.

4. Balancing of different masses rotating in different planes.

We shall now discuss these cases, in detail, in the following pages.

Types of balancing:

a) Static Balancing: i) Static balancing is a balance of forces due to action of gravity.

ii) A body is said to be instatic balance when its centre of gravity is in the axis of rotation.

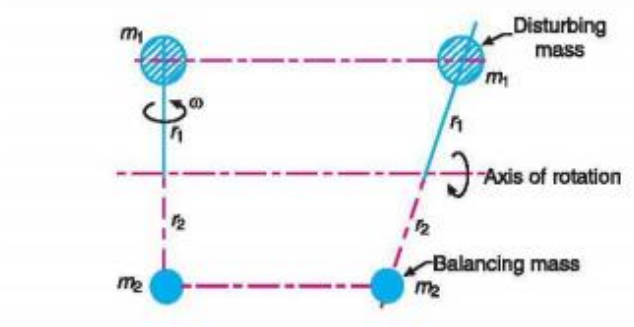
b) Dynamic balancing: i) Dynamic balance is a balance due to the action of inertia forces. ii) A body is said to be in dynamic balance when the resultant moments or couples, which involved in the acceleration of different moving parts is equal to zero. iii) The conditions of dynamic balance are conditions of static balance.

Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane:

Consider a disturbing mass m1 attached to a shaft rotating at □ rad/s as shown in Fig. Let r1 be the radius of rotation of the mass m1 (i.e. distance between the axis of rotation of the shaft and the centre of gravity of the mass m1).

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m2) may be attached in the same plane of rotation as that of disturbing mass (m1) such that the centrifugal forces due to the two masses are equal and opposite.



Let r_2 = Radius of rotation of the balancing mass m_2 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

∴ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \qquad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_1 \cdot \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes:

We have discussed in the previous article that by introducing a single balancing mass in the same plane of rotation as that of disturbing mass, the centrifugal forces are balanced. In other words, the two forces are equal in magnitude and opposite in direction. But this type of arrange- ment for balancing gives rise to a couple which tends to rock the shaft in its bearings. Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for static balancing.

2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the

Algebraic sum of the moments about any point in the plane must be zero. The conditions (1) and (2) together give dynamic balancing. The following two possibilities may arise while attaching the two balancing masses :

1. The plane of the disturbing mass may be in between the planes of the two balancing masses, and

2. The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

We shall now discuss both the above cases one by one.

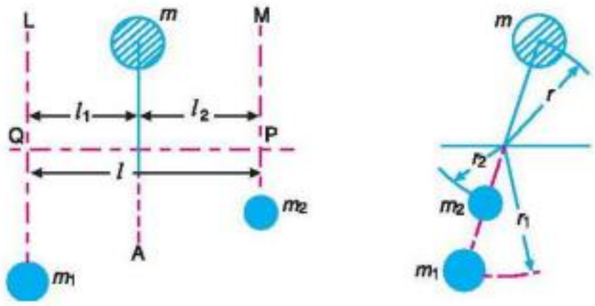
1. When the plane of the disturbing mass lies in between the planes of the two balancing masses

Consider a disturbing mass m lying in a plane A to be balanced by two rotating masses m1 and m2 lying in two different planes L and M as shown in Fig. 21.2. Let r, r1 and r2 be the radii of rotation of the masses in planes A, L and M respectively.

Let l_1 = Distance between the planes A and L,

l_2 = Distance between the planes A and M, an

l = Distance between the planes L and M.



We know that the centrifugal force exerted by the mass m in the plane A,

$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane L,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane M,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C = F_{C1} + F_{C2} \quad \text{or} \quad m \cdot \omega^2 \cdot r = m_1 . \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\therefore \quad m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \qquad \ldots (i)$$

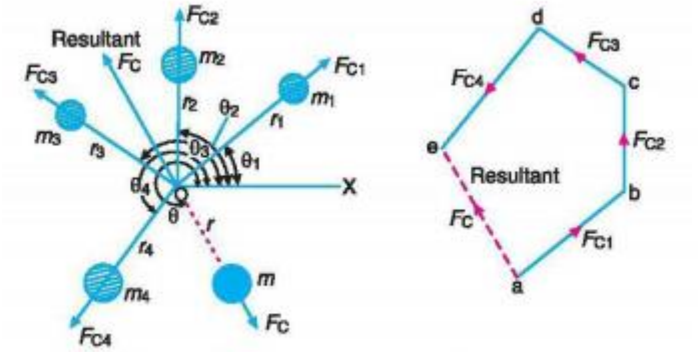
Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

Balancing of Several Masses Rotating in the Same Plane:

Consider any number of masses (say four) of magnitude m1, m2, m3 and m4 at distances of r1, r2, r3 and r4 from the axis of the rotating shaft. Let □1, □2 , □3 and □4 be the angles of these masses with the horizontal line OX, as shown in Fig. 21.4 (a). Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of rad/s

The magnitude and position of the balancing mass may be found out analytically or graphically asdiscussed below.



(a) Space diagram. (b) Vector diagram.

1. Analytical method

The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below

1. First of all, find out the centrifugal force* (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.

2. Resolve the centrifugal forces horizontally and vertically and find their sums, i.e.∑ H and ∑V . We know that

Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in ***opposite direction.***

6. Now find out the magnitude of the balancing mass, such that

$$F_C = m \cdot r$$

where m = Balancing mass, and

r = Its radius of rotation.

2. Graphical method

The magnitude and position of the balancing mass may also be obtained graphically as discussed below :

1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig. (a).

2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.

3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that ab represents the centrifugal force exerted by the mass m1 (or m1.r1) in magnitude and direction to some suitable scale. Similarly, draw bc, cd and de to represent centrifugal forces of other masses m2, m3 and m4 (or m2.r2, m3.r3 and m4.r4).

4. Now, as per polygon law of forces, the closing side ae represents the resultant force in magnitude and direction, as shown in Fig. (b).

5. The balancing force is, then, equal to the resultant force, but in opposite direction.

6. Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

$$m \cdot \omega^2 \cdot r = \text{Resultant centrifugal force}$$

$$m.r = \text{Resultant of } m_1.r_1,\ m_2.r_2,\ m_3.r_3 \text{ and } m_4.r_4$$

Balancing of Several Masses Rotating in Different Planes:

When several masses revolve in different planes, they may be transferred to a reference plane (briefly written as R.P.), which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane. In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied:

1. The forces in the reference plane must balance, i.e. the resultant force must be zero.

2. The couples about the reference plane must balance, i.e. the resultant couple must be zero.

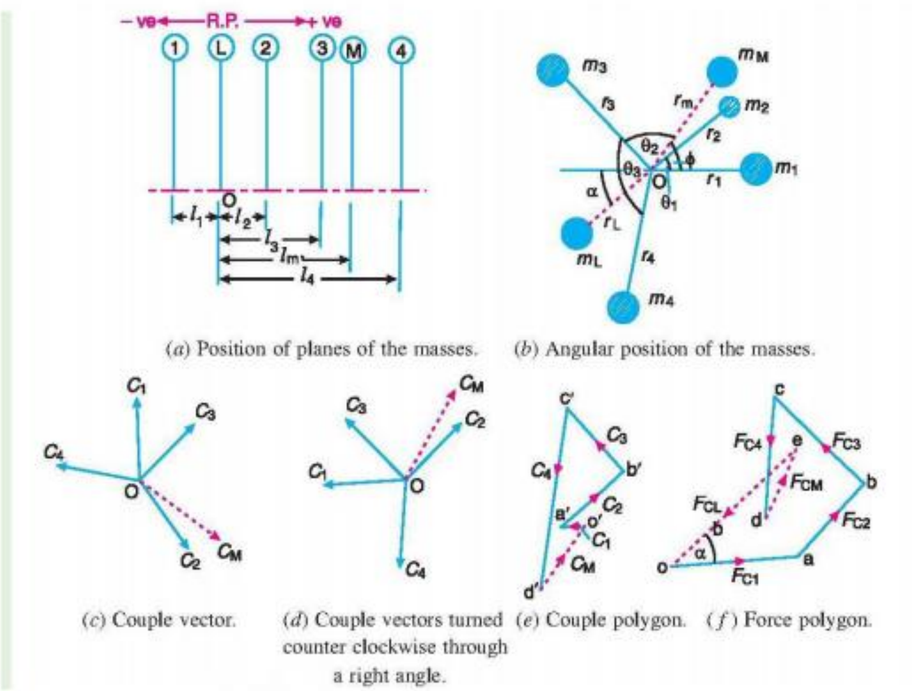
Let us now consider four masses m1, m2, m3 and m4 revolving in planes 1, 2, 3 and 4 respectively as shown in

Fig. (a). The relative angular positions of these masses are shown in the end view [Fig. (b)]. The magnitude of the balancing masses mL and mM in planes L and M may be obtained as discussed below :

1. Take one of the planes; say L as the reference plane (R.P.). The distances of all the other planes to the left of the reference plane may be regarded as negative, and those to the right as positive.

2. Tabulate the data as shown in Table 21.1. The planes are tabulated in the same order in which they occur, reading from left to right.

Plane (1)	Mass (m) (2)	Radius(r) (3)	Cent.force $\div \omega^2$ (m.r) (4)	Distance from Plane L (l) (5)	Couple $\div \omega^2$ (m.r.l) (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



(a) Position of planes of the masses. (b) Angular position of the masses.

(c) Couple vector. (d) Couple vectors turned counter clockwise through a right angle. (e) Couple polygon. (f) Force polygon.

3. A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by

Transferring m1 to the reference plane through O is proportional to m1.r1.l1 and acts in a plane through Om1 and perpendicular to the paper. The vector representing this couple is drawn in the plane of the paper and perpendicular to Om1 as shown by OC1 in Fig. 21.7 (c). Similarly, the vectors OC2, OC3 and OC4 are drawn perpendicular to Om2, Om3 and Om4 respectively and in the plane of the paper.

4. The couple vectors as discussed above, are turned counter clockwise through a right angle for convenience of drawing as shown in Fig. 21.7 (d). We see that their relative positions remain unaffected. Now the vectors OC2, OC3 and OC4 are parallel and in the same direction as Om2, Om3 and Om4, while the vector OC1 is parallel to Om1 but in *opposite direction. Hence the couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane.

5. Now draw the couple polygon as shown in Fig. 21.7 (e). The vector d2 o2 represents the balanced couple. Since the balanced couple CM is proportional to mM, rM, lM, therefore

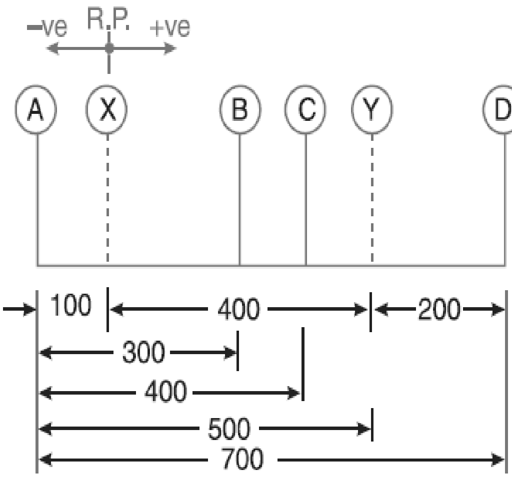
$$m_L \cdot r_L = \text{vector } eo \quad \text{or} \quad m_L = \frac{\text{vector } eo}{r_L}$$

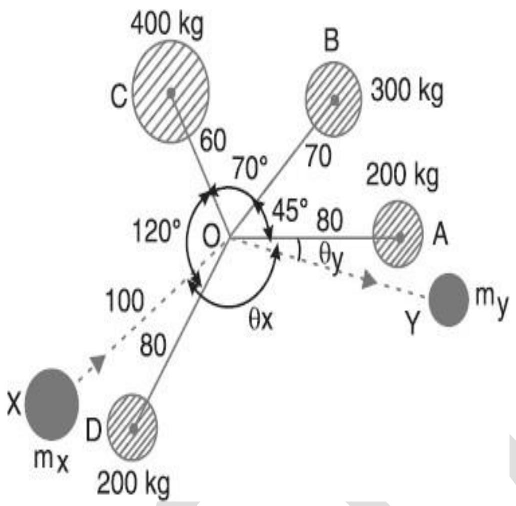
From this expression, the value of the balancing mass mL in the plane L may be obtained and the angle of inclination of this mass with the horizontal may be measured from Fig.(b).

PROBLEMS:

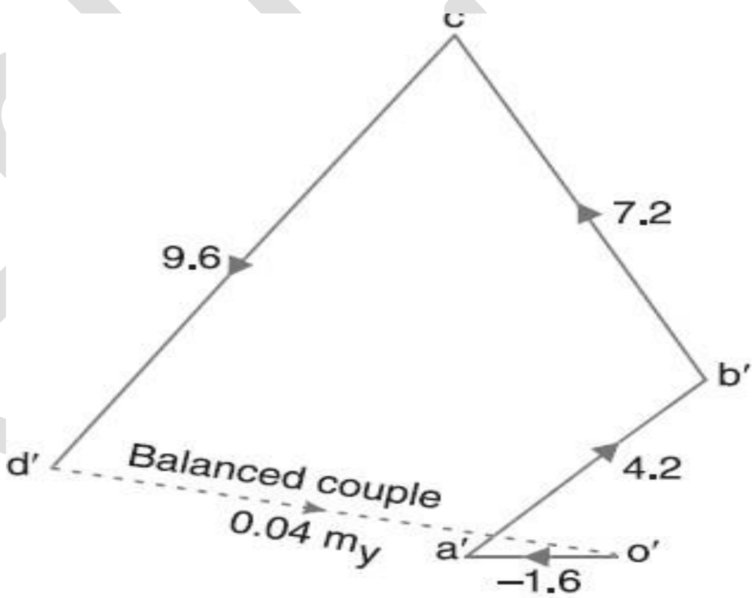
1. A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

Given : **m_A = 200 kg ; m_B = 300 kg ; m_C = 400 kg ;m_D = 200 kg ,r_A = 80 mmv= 0.08m ; r_B = 70 mm = 0.07 m ; r_C = 60 mm = 0.06 m ; r_D = 80 mm = 0.08 m ; r_X = r_Y = 100 mm = 0.1 m**





Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent.force ÷ ω^2 (m.r) kg-m (4)	Distance from Plane x(l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A	200	0.08	16	– 0.1	– 1.6
X(*R.P.*)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

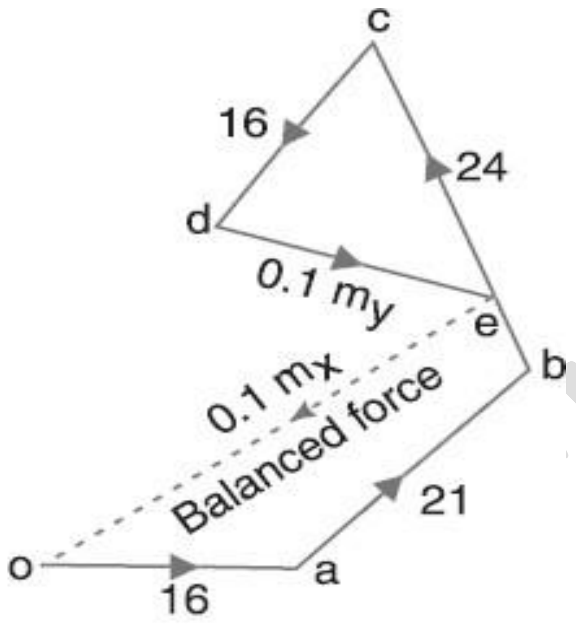


(*c*) Couple polygon.

By measurement, the angular position of m_Y is $\theta_Y = 12°$ in the clockwise direction from mass m_A (*i.e.* 200 kg).

$$0.04\, m_Y = \text{vector } d'\, o' = 7.3 \text{ kg-m}^2$$

$$m_Y = 182.5 \text{ kg}$$



(*d*) Force polygon.

$$0.1\, m_X = \text{vector } eo = 35.5 \text{ kg-m}$$

$$m_X = 355 \text{ kg}$$

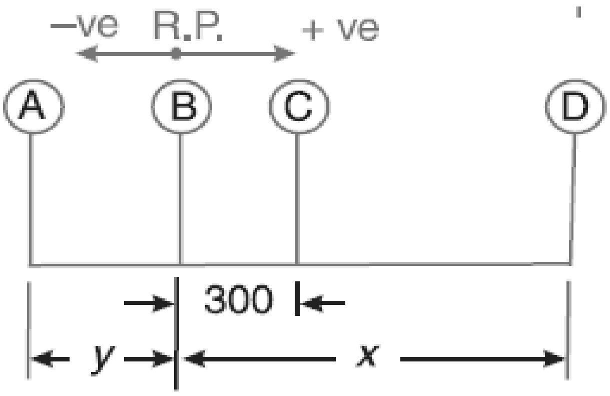
By measurement, the angular position of m_X is $\theta_X = 145°$ in the clockwise direction from mass m_A (*i.e.* 200 kg).

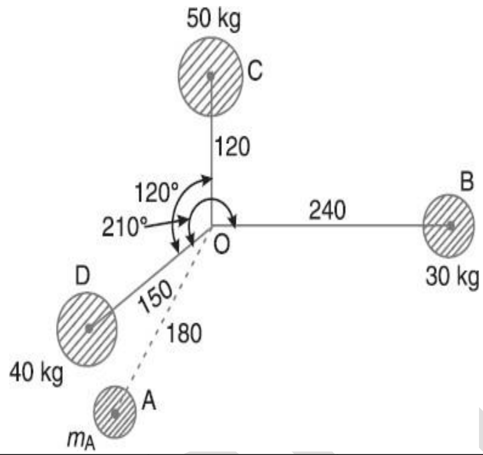
2. Four masses A, B, C and D as shown below are to be completely balanced. The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

1. The magnitude and the angular position of mass A ; and
2. The position of planes A and D.

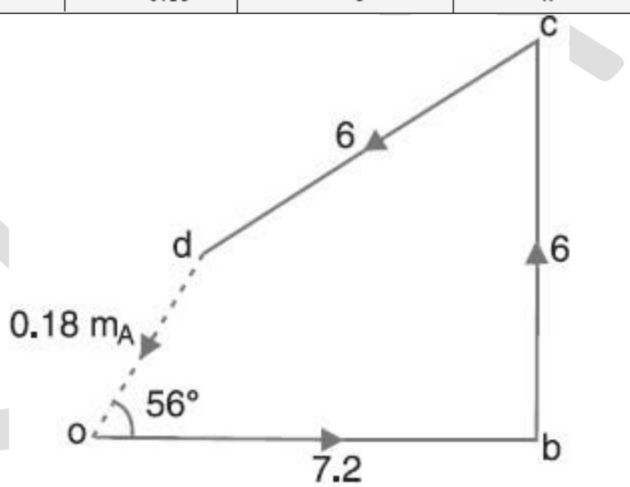
Given r_A = 180 mm = 0.18 m ; m_B = 30 kg ; r_B = 240 mm = 0.24 m ; m_C = 50 kg ; r_C = 120 mm = 0.12 m ; m_D = 40 kg ; r_D = 150 mm = 0.15 m ; ∠BOC = 90° ; ∠BOD = 210° ; ∠COD = 120°

	A	*B*	*C*	*D*
Mass (kg)	—	*30*	*50*	*40*
Radius (mm)	*180*	*240*	*120*	*150*





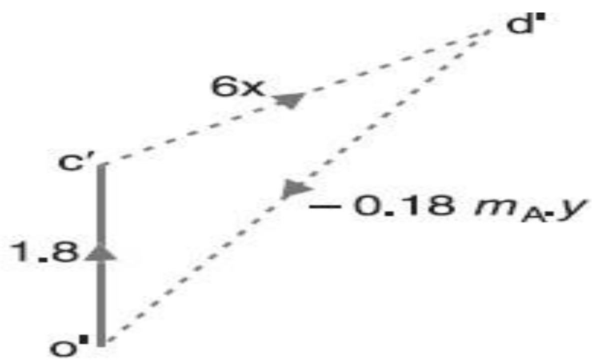
Plane *(1)*	*Mass* *(m) kg* *(2)*	*Radius* *(r) m* *(3)*	*Cent.force* $\div \omega^2$ *(m.r) kg-m* *(4)*	*Distance from plane B (l) m* *(5)*	*Couple* $\div \omega^2$ *(m.r.l) kg-m²* *(6)*
A	m_A	0.18	$0.08\ m_A$	$-y$	$-0.18\ m_A.y$
B (R.P)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	$6x$



(*c*) Force polygon.

$0.18\ m_A$ = Vector do = 3.6 kg-m $m_A = 20$ kg

the angular position of mass *A* from mass *B* in the anticlockwise direction is $\angle AOB = 236°$



(*d*) Couple polygon.

$$6\,x = \text{vector } c'd' = 2.3 \text{ kg-m}^2$$

$$x = 0.383 \text{ m}$$

$$-\,0.18\, m_A . y = \text{vector } o'd' = 3.6 \text{ kg-m}^2$$

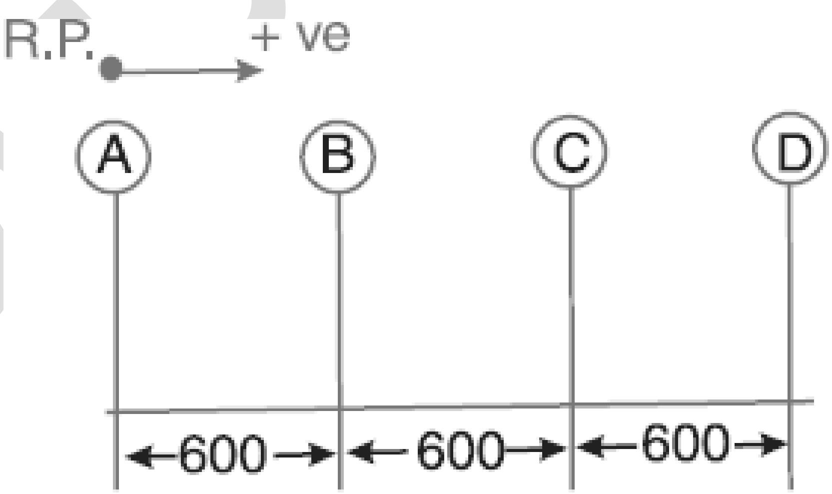
$$-\,0.18 \times 20\, y = 3.6$$

$$y = -\,1 \text{ m}$$

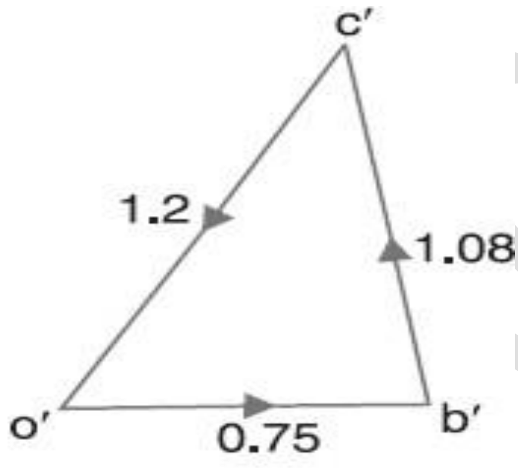
The negative sign indicates that the plane *A* is not towards left of *B* as assumed but it is 1000 mm towards right of plane *B*.

3. A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively. Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

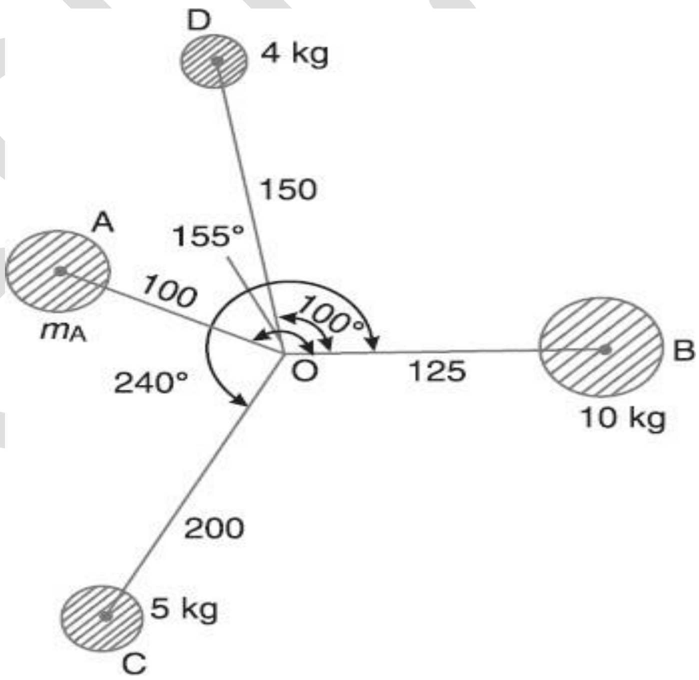
Given: r_A = 100 mm = 0.1 m ; r_B = 125 mm = 0.125 m ; r_C = 200 mm = 0.2 m ;
r_D = 150 mm = 0.15 m ; m_B = 10 kg ; m_C = 5 kg ; m_D = 4 kg



Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. Force ÷ ω^2 (m.r)kg-m (4)	Distance from plane A (l)m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

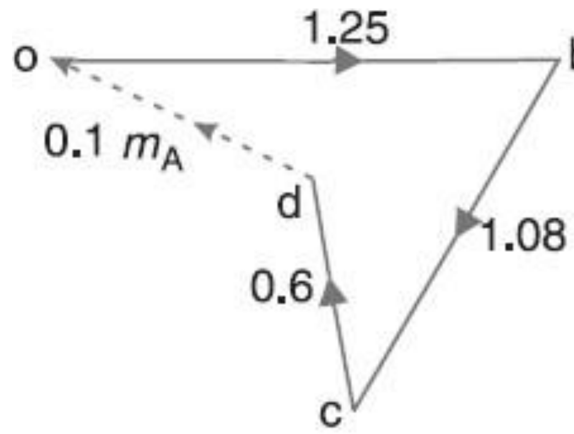


(c) Couple polygon.



$$\angle BOC = 240°$$

$$\angle BOD = 100°$$



(*d*) Force polygon.

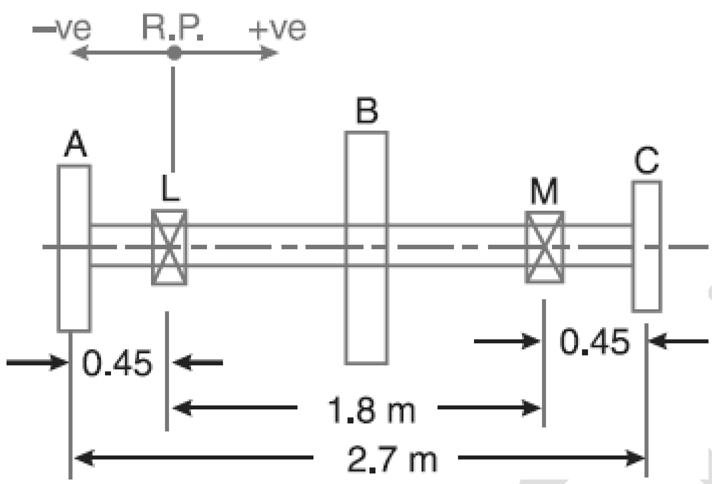
$$0.1\ m_A = 0.7 \text{ kg-m}^2$$

$$m_A = 7 \text{ kg}$$

$$\angle BOA = 155°$$

4. A shaft is supported in bearings 1.8 m apart and projects 0.45 m beyond bearings at each end. The shaft carries three pulleys one at each end and one at the middle of its length. The mass of end pulleys is 48 kg and 20 kg and their centre of gravity are 15 mm and 12.5 mm respectively from the shaft axis. The centre pulley has a mass of 56 kg and its centre of gravity is 15 mm from the shaft axis. If the pulleys are arranged so as to give static balance, determine: 1. relative angular positions of the pulleys, and 2. dynamic forces produced on the bearings when the shaft rotates at 300 r.p.m.

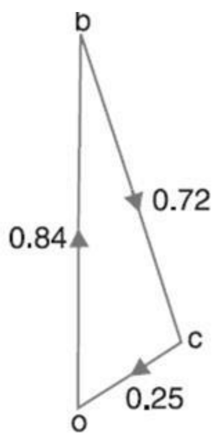
Given : m_A = 48 kg ; m_C = 20 kg ; r_A = 15 mm = 0.015 m ; r_C = 12.5 mm = 0.0125 m ; m_B = 56 kg ; r_B = 15 mm = 0.015 m ; N = 300 r.p.m. or ω= 2 π × 300/60 = 31.42 rad/s



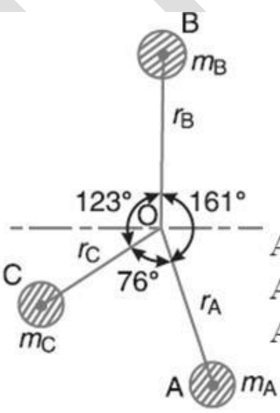
(a) Position of shaft and pulleys.

1. *Relative angular position of the pulleys*

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force ÷ ω^2 (m.r) kg-m* (4)	*Distance from plane L(l)m* (5)	*Couple ÷ ω^2 (m.r.l) kg-m^2* (6)
A	48	0.015	0.72	– 0.45	– 0.324
L(R.P)	m_L	r_L	$m_L.r_L$	0	0
B	56	0.015	0.84	0.9	0.756
M	m_M	r_M	$m_M.r_M$	1.8	1.8 $m_M.r_M$
C	20	0.0125	0.25	2.25	0.5625

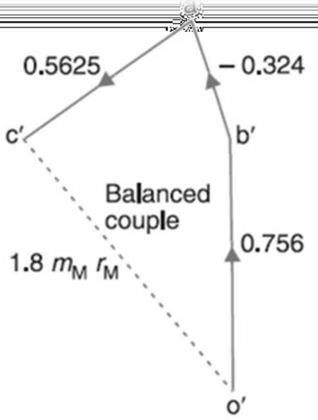


(c) Force polygon



Angle between pulleys *B* and *A* = 161°
Angle between pulleys *A* and *C* = 76°
Angle between pulleys *C* and *B* = 123°

(b) Angular position of pulleys.



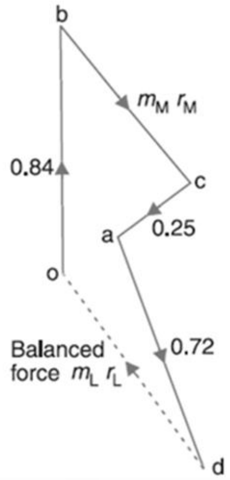
(d) Couple polygon.

$$1.8\, m_M . r_M = \text{vector } c'o' = 0.97 \text{ kg-m}^2$$

$$m_M . r_M = 0.54 \text{ kg-m}^2$$

Dynamic force at the bearing M

$$= m_M . r_M . \omega^2 = 0.54\ (31.42)^2 = 533 \text{ N}$$



$$m_L . r_L = 0.54 \text{ kg-m}$$

Dynamic force at the bearing L

$$= m_L . r_L . \omega^2 = 0.54\ (31.42)^2 = 533 \text{ N}$$

MODULE-4

GOVERNERS

Introduction:

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load e.g. when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits. A little consideration will show, that when the load increases, the configuration of the governor changes and a valve is moved to increase the supply of the working fluid ; conversely, when the load decreases, the engine speed increases and the governor decreases the supply of working fluid.

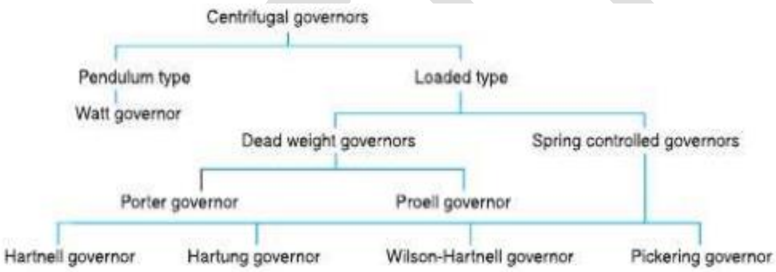
Note: We have discussed in Chapter 16 (Art. 16.8) that the func- tion of a flywheel in an engine is entirely different from that of a governor. It controls the speed variation caused by the fluctuations of the engine turning moment during each cycle of operation. It does not control the speed variations caused by a varying load. The varying demand for power is met by the governor regulating the supply of working fluid.

Types of Governors:

The governors may, broadly, be classified as

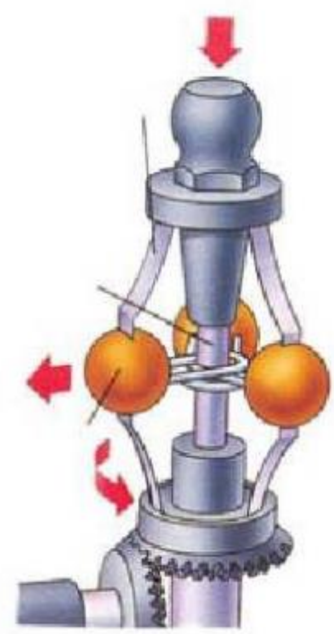
1. Centrifugal governors, and

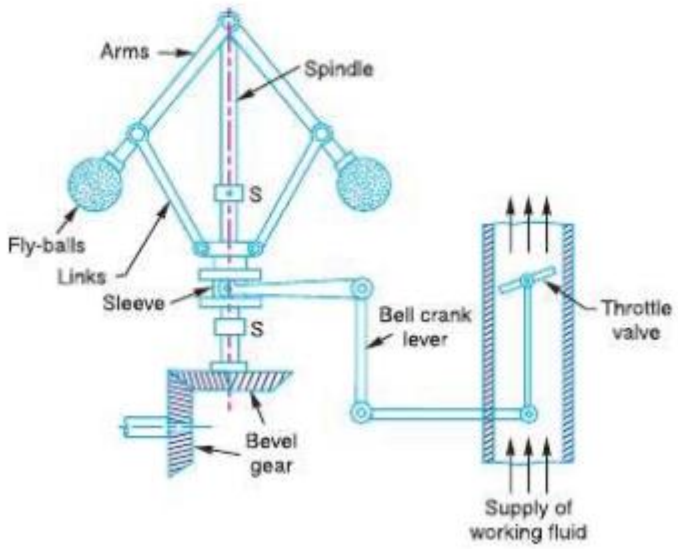
2. Inertia governors.



Centrifugal governor:

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the controlling force*.It consists of two balls of equal mass, which are attached to the arms as shown in Fig. 18.1. These balls are known as governor balls or fly balls. The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted tothe spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle ;but can slide up and down. The balls and the sleeve rises when thespindle speed increases, and falls when the speed decreases. In order tolimit the travel of thesleeve in upward and downward directions, twostops S, S are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls. When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced.





Terms Used in Governors:

The following terms used in governors are important from the subject point of view ;

1. Height of a governor. It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by h.

2. Equilibrium speed. It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

3. Mean equilibrium speed. It is the speed at the mean position of the balls or the sleeve.

4. Maximum and minimum equilibrium speeds. The speeds at the maximum and minimum radius of rotationof the balls, without tending to move either way are known as maximum and mini- mum equilibrium speeds respectively.

Note : There can be many equilibrium speeds between the mean and the maximum and the mean and the mini- mum equilibrium speeds

5. Sleeve lift. It is the vertical distance which the sleeve travels due to change in equilibrium speed.

Gravity loaded controlled governors:

Porter governors:

The type of governor, which is illustrated at fig-3.1 (b), is known as the Porter governor. The only respect in which it differs from the Watt governor is in the use of a heavily weighted sleeve. The additional downward force increases the speed of revolution required to enable the balls to rise to any pre-determined level.

Let 'w' be the weight of each ball and 'W' be the weight of the central load. T_1 be the tension in the upper arm and T_2 the tension in the suspension link. α and β be the inclinations to the vertical of the upper arm and suspension links respectively. The weight of arms and weight of suspension links and the effect of friction to the movement of the sleeve are neglected.

There are several ways of determining the relation between the height 'h' and the speed 'ω'. In this chapter, two methods are used to derive the relation.

(i) Instantaneous Center Method

Consider the equilibrium of the forces acting on the suspension link 'AC', which is shown in fig-3.4. These forces are 'F, w and T_1 at C and $\frac{W}{2}$ and Q at A. The equation connecting 'F, w and W' is derived by taking moment about I, the point of intersection of the lines of action of forces T_1 and Q. This point of intersection I is also the instantaneous center of the link AC. The point I lies at the point of intersection of BC produce and a line drawn through A perpendicular to the axis of the governor spindle.

Taking moment about I,

$$F \times CD = w \times ID + \frac{W}{2}(ID + DA)$$

$$F = w \times \frac{ID}{CD} + \frac{W}{2}\left(\frac{ID}{CD} + \frac{DA}{CD}\right)$$

$$= w\tan\alpha + \frac{W}{2}(\tan\alpha + \tan\beta)$$

$$= \left\{\frac{W}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right) + w\right\}\tan\alpha$$

$$= \left\{\frac{W}{2}(1 + k) + w\right\}\tan\alpha$$

where $k = \frac{\tan\beta}{\tan\alpha}$.

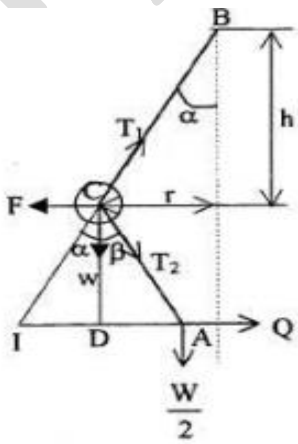


Fig-3.4

If 'h' be the height of the governor, then $\tan\alpha = \frac{r}{h}$. Further, we have $F = \frac{w}{g}\omega^2 r$.

Therefore, we get

$$\frac{w}{g}\omega^2 r = \left[\frac{W}{2}(1+k) + w\right]\frac{r}{h} \qquad \text{(or)}$$

$$\omega^2 = \left[\frac{\frac{W}{2}(1+k) + w}{w}\right]\frac{g}{h} \qquad \text{(i)}$$

When the length of the arms and the suspension links are of equal length and the axis of the joints at B and A either intersect the governor spindle or are at equal distances from the governor spindle the value 'k' is equal to 1 and the equation (i) reduces to the form

$$\omega^2 = \left(\frac{W+w}{w}\right)\frac{g}{h} \qquad \text{(ii)}$$

When the lengths of the arms are unequal and the axes of the joints at B and A are at different distances from the governor spindle the k will have a different value for each radius of rotation of the governor balls, This value of 'k' can be best found by calculating the value of α and β. It should be noted that when 'k' is not equal to 1, its value changes as the height of the governor changes.

For the simple Watt governor, the weight of the sleeve W is negligible and we have either from equation (i) or (ii) the relation $\omega^2 = \frac{g}{h}$ which has derived earlier.

(ii) Equilibrium Method

The governor sleeve, which is loaded by the weight W is in equilibrium under a system of three forces, W the load on the sleeve and the tensions T_2 in the two lowered suspension links. As the system of forces is in equilibrium, the force triangle drawn for these forces must be a closed one as shown in fig-3.5 (a).

The pin joint C between the upper arm and the lower suspension link must be in equilibrium under the action of the four forces as under:

(i) The weight of the ball 'w'

(ii) Radially outwards acting centrifugal force $F = \frac{w}{g}\omega^2 r$

(iii) Tension T_1 in the upper arm

(iv) Tension T_2 in the lower suspension link.

These four forces must form a closed polygon as shown in fig-3.5 (b).

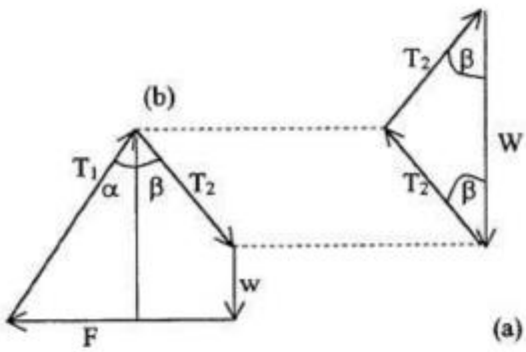


Fig-3.5

From force triangle for the sleeve, we get

$W = 2\,T_2 \cos\beta$ (or) $T_2 = \dfrac{W}{2\cos\beta}$ (iii)

From the polygon of forces on the ball, we have

$T_1 \cos\alpha = T_2 \cos\beta + w$ (resolving vertically) (iv)

Resolving horizontally,

$F = T_1 \sin\alpha + T_2 \sin\beta$ (v)

From equation (iv)

$$T_1 = \frac{\frac{W}{2} + w}{\cos\alpha}$$

When the value of T_1 and T_2 are substituted in the equation (v),

From equation (iv)

$$T_1 = \frac{\frac{W}{2} + w}{\cos\alpha}$$

When the value of T_1 and T_2 are substituted in the equation (v),

$$F = \left(\frac{W}{2} + w\right)\tan\alpha + \frac{W}{2}\tan\beta$$

$$= \left[\frac{W}{2}(1+k) + w\right]\tan\alpha \quad \text{(v)}$$

where $k = \dfrac{\tan\beta}{\tan\alpha}$.

By substituting the value of $\tan\alpha$ and F, equation (i), which is derived earlier, can be done.

Spring loaded controlled governor's:

3.5 Spring Loaded Controlled Governors

In spring loaded controlled governors the control of speed is affected either wholly or in part by means of springs. Some of the representative of spring loaded controlled governors are shown in fig-3.2.

The spring loaded controlled governors posses the following advantages over the gravity loaded controlled governors.

(i) The spring loaded controlled governors may be operated at very high speeds.

(ii) With proper proportioning the spring loaded controlled governors can be made both powerful and capable of very closed regulation.

(iii) It can be much smaller in over all size.

(iv) As it does not depend on gravity for its action, it may revolve about a horizontal, vertical or inclined axis.

In spring loaded controlled governors the spring may be placed upon the axis of rotation or they may be transverse as shown in fig-3.2.

(a) Spring loaded Controlled Governor of the Hartnell Type

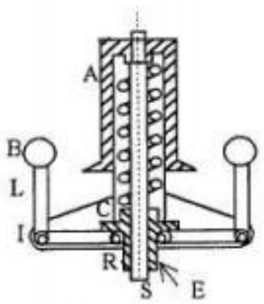


Fig-3.10

Fig-3.10 shows spring loaded controlled governor of Hartnell type. Two bell crank levers L are mounted on pins I, carried by the frame A, which is attached to the rotating spindle S. Each lever carries a ball B at the end of one arm and a roller R the end of the other. The centrifugal forces of the balls cause the rollers R to press against the collar C on the sleeve E. The upward pressure of the rollers on the collar of the sleeve is balanced by the downward thrust of the helical spring, which is in compression. The angle of the bell crank lever is usually 90° but in practice it may be grater.

Let w be the weight of each ball, S the spring force exerted on the sleeve, k the stiffness of the spring, ω the speed of rotation, r the radius of rotation, a and b the lengths of the vertical and horizontal arms of the bell crank lever and F the centrifugal force on the ball.

By taking moment about the fulcrum of the lever, neglecting the effect of pull of gravity on the governor balls and arms,

$$F \times a = \frac{S}{2} \times b$$

or $$S = 2F\frac{a}{b} \qquad \text{(i)}$$

It is assumed that the arms are mutually perpendicular and the lines of action of forces are at right angles to the arm.

Let the suffixes 1 and 2 denote the values of maximum and minimum radii respectively. Then at maximum radius

$$S_1 = 2F_1\frac{a}{b} \qquad \text{(ii)}$$

At minimum radius, $$S_2 = 2F_2\frac{a}{b} \qquad \text{(iii)}$$

$$\therefore S_1 - S_2 = 2\frac{a}{b}(F_1 - F_2)$$

Let θ be the angular movement of the bell crank lever from the position of minimum radius to the position of the maximum radius, then

$$(r_1 - r_2) = a\theta \qquad \text{(iv)}$$

If h be the lift of the sleeve, then

$$h = b\theta \qquad \text{(v)}$$

Dividing equation (v) by (iv),

$$\frac{h}{r_r - r_2} = \frac{b}{a} \qquad \text{(or)}$$

$$h = \frac{b}{a}(r_1 - r_2) \qquad \text{(vi)}$$

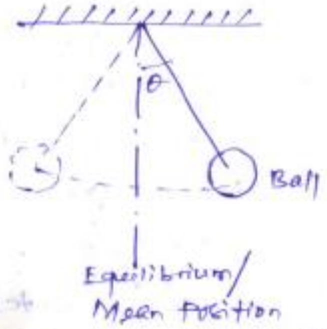
The difference in the forces exerted by the compressed spring in the two positions is $S_1 - S_2$; therefore, the force per unit compression is known as the stiffness of the spring. The stiffness of the spring is denoted by k.

MODULE-05

VIBRATIONS & GYROSCOPE

Basic Concept of Vibration / What is Vibration?

When body particles are displaced by the application of external force, the internal forces in the form of elastic energy are present in the body. These forces try to bring the body to its original position. At equilibrium position, the entire elastic energy is converted into kinetic energy and the body continues to move in the opposite direction and the process repeats.



(Fig-1 A simple pendulum)

– So any motion which repeats itself after an interval of time is called vibration; e.g. simple pendulum (shown in fig-1)

Reasons of Vibrations:-

1. Unbalanced forces in the machine :- forces produced within the machine
2. Dry friction between two mating surfaces:- This produces a self excited vibration
3. External excitations:- The excitations may be periodic, random etc.
4. Earthquake: Responsible for failure of buildings/ dams etc.
5. Wind:- It may cause vibration of transmission and telephone lines under certain condition.

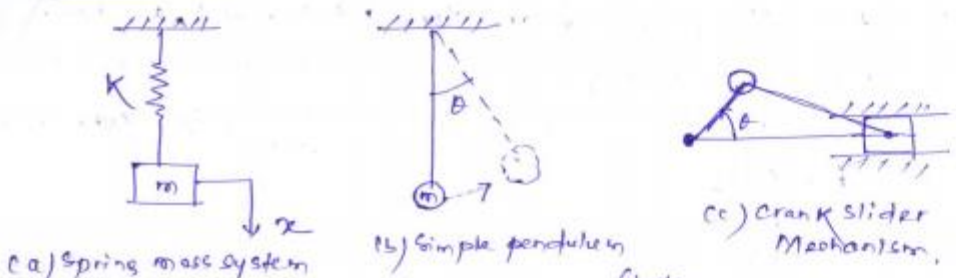
Definitions:-

1. Periodic motion → A motion which repeats itself after equal interval of time.
2. Time period → Time taken to complete one cycle.
3. Frequency → No of cycles/unit time
4. Simple Harmonic Motion → A periodic motion of a particle whose acceleration is always directed towards the mean position.
5. Amplitude of motion → Maximum displacement of a vibrating body from mean position
6. Free vibrations → Vibration of a system because of its own elastic property without any external exciting forces acting on it.
7. Forced vibration → The vibrations the system executes under the action of an external periodic force. The frequency of vibration is same to that of excitation.
8. Natural frequency → Frequency of free vibration of the system. It is constant for a given system.
9. Resonance :→ Vibration of a system ~~when~~ in which the frequency of external force is equal to the natural frequency of the system.
10. Damping → Resistance to the motion of the vibrating body.
11. Degree of freedom → No of independent coordinates required to specify completely the configuration of the system at any instant.

Few examples of Single degree of freedom system, have been

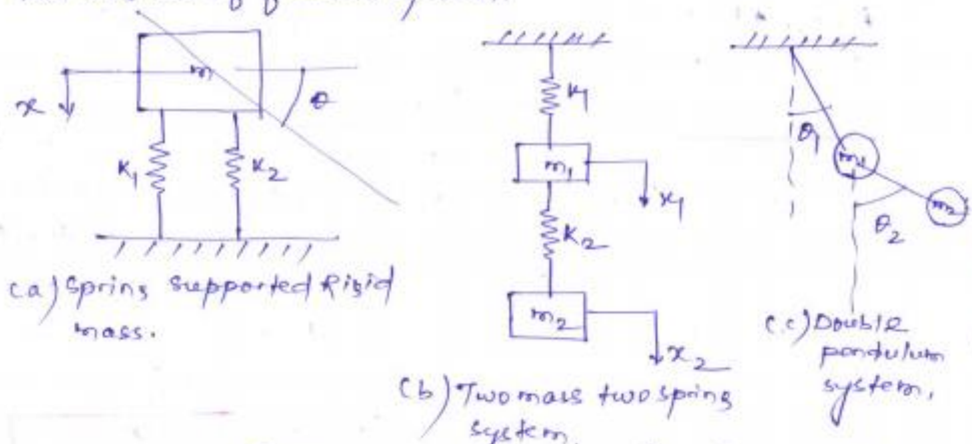
③

Example :-



(Fig 2:- Examples of Single Degree of Freedom system.)

shown in Fig.(2). And Fig(3) depicts few examples of two degrees of freedom system.



[Fig-3 :- Examples of two degrees of freedom (DOF) systems]

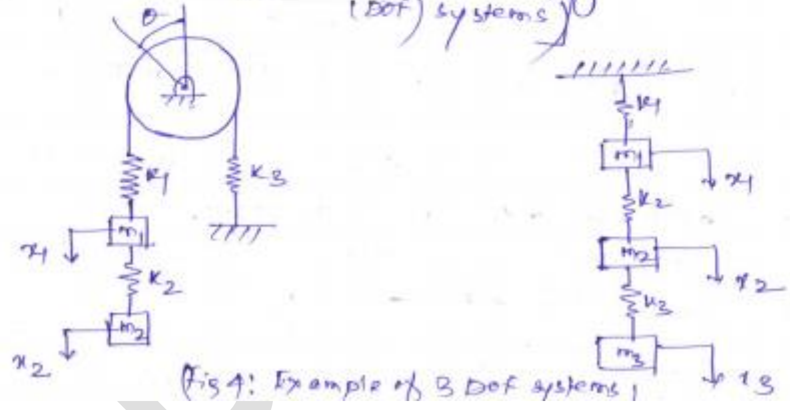
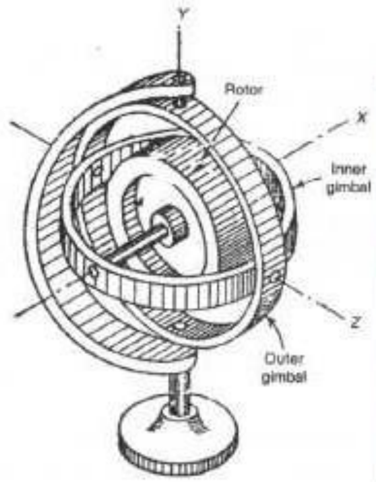


Fig 4: Example of 3 DOF systems.

Introduction

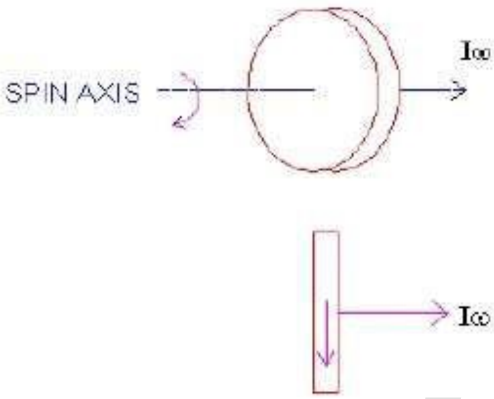
'Gyre' is a Greek word, meaning 'circular motion' and Gyration means the whirling motion. A gyroscope is a spatial mechanism which is generally employed for the study of precessional motion of a rotary body. Gyroscope finds applications in gyrocompass, used in aircraft, naval ship, control system of missiles and space shuttle. The gyroscopic effect is also felt on the automotive vehicles while negotiating a turn.

A gyroscope consists of a rotor mounted in the inner gimbal. The inner gimbal is mounted in the outer gimbal which itself is mounted on a fixed frame as shown in Fig.. When the rotor spins about X-axis with angular velocity ω rad/s and the inner gimbal precesses (rotates) about Y-axis, the spatial mechanism is forced to turn about Z-axis other than its own axis of rotation, and the gyroscopic effect is thus setup. The resistance to this motion is called gyroscopic effect.



ANGULAR MOTION

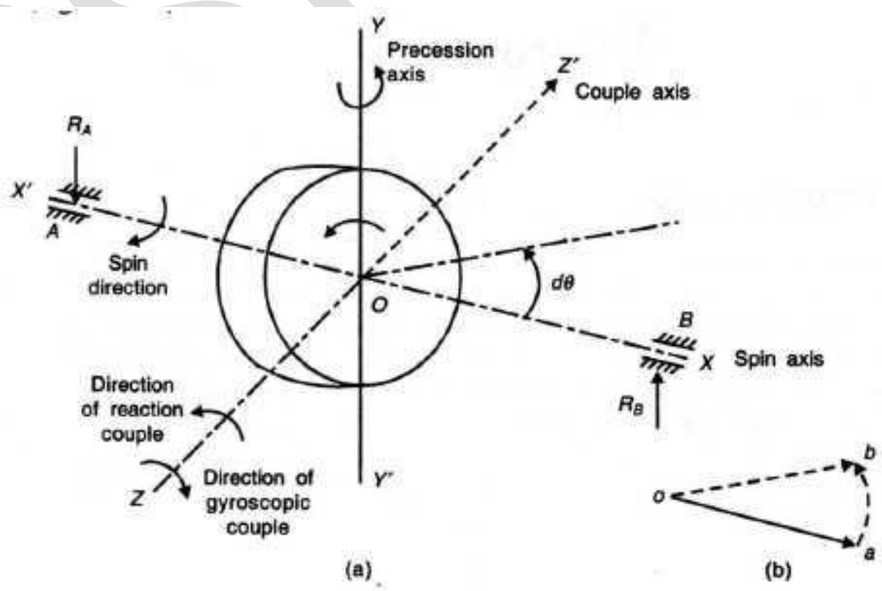
A rigid body, (Fig.) spinning at a constant angular velocity ω rad/s about a spin axis through the mass centre. The angular momentum 'H' of the spinning body is represented by a **vector** whose magnitude is 'Iω'. I represents the mass amount of inertia of the rotor about the axis of spin.



The direction of the angular momentum can be found from the right hand screw rule or the right hand thumb rule. Accordingly, if the fingers of the right hand are bent in the direction of rotation of rotor, then the thumb indicates the direction of momentum.

GYROSCOPIC COUPLE

Consider a rotary body of mass *m* having radius of gyration *k* mounted on the shaft supported at two bearings. Let the rotor spins (rotates) about X-axis with constant angular velocity ω rad/s. The X-axis is, therefore, called spin axis, Y-axis, precession axis and Z-axis, the couple or torque axis (Fig.).



The angular momentum of the rotating mass is given by,

$$H = mk^2 \omega = I\omega$$

Now, suppose the shaft axis (X-axis) precesses through a small angle $\delta\theta$ about Y-axis in the plane *XOZ*, then the angular momentum varies from *H* to *H* + δ*H*, where δ*H* is the change in the angular momentum, represented by vector ab [Figure 15.2(b)]. For the small value of angle of rotation 50, we can write

$$ab = oa \times \delta\theta$$
$$\delta H = H \times \delta\theta$$
$$= I\omega\delta\theta$$

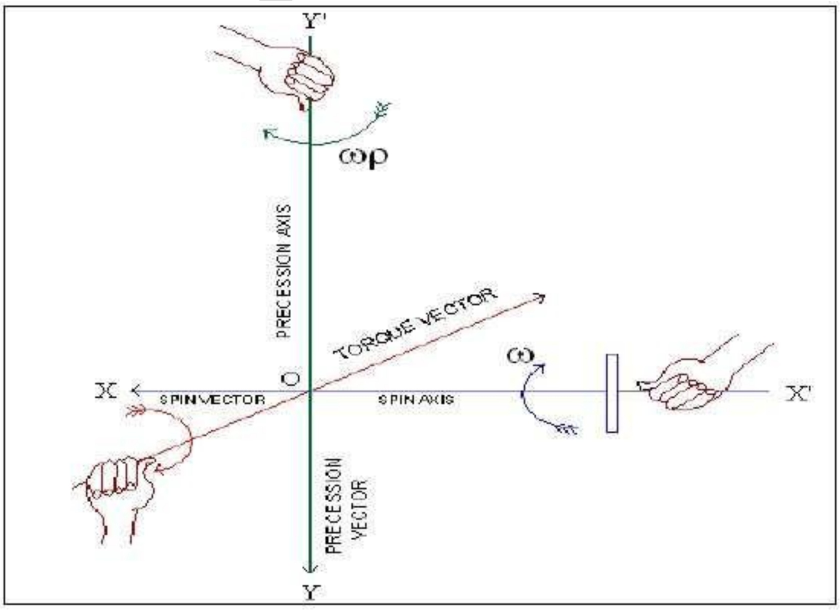
However, the rate of change of angular momentum is:

$$C = \frac{dH}{dt} = \lim_{\delta t \to 0}\left(\frac{I\omega\delta\theta}{\delta t}\right)$$
$$= I\omega\frac{d\theta}{dt}$$

$$\mathbf{C = I\omega\omega_p}$$

Direction of Spin vector, Precession vector and Couple/Torque vector with forced precession

To determine the direction of spin, precession and torque/couple vector, right hand screw rule or right hand rule is used. The fingers represent the rotation of the disc and the thumb shows the direction of the spin, precession and torque vector (Fig.).

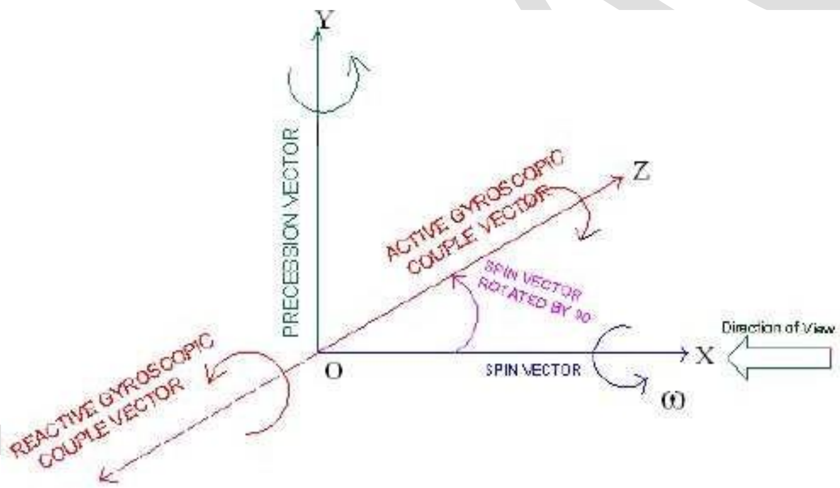


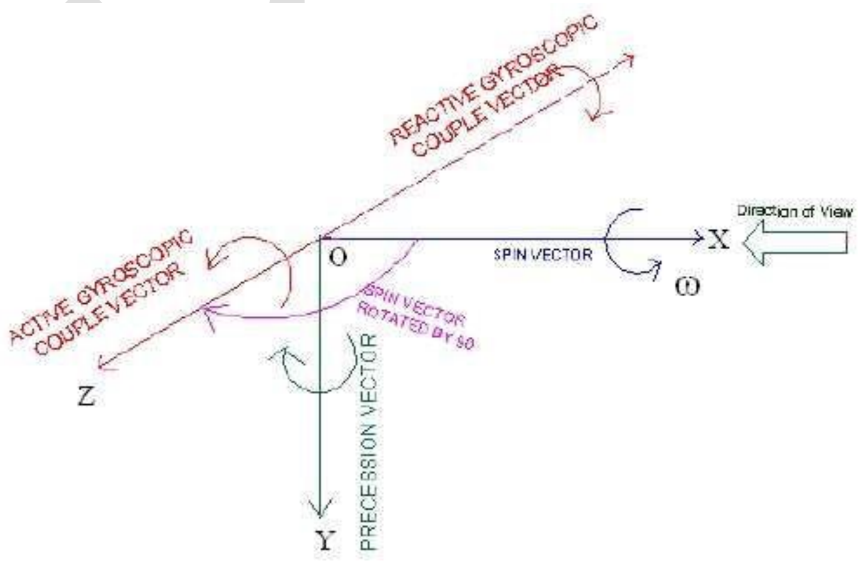
The method of determining the direction of couple/torque vector is as follows

Case (i):

Consider a rotor rotating in anticlockwise direction when seen from the right (Fig.5 and Fig. 6), and to precess the spin axis about precession axis in clockwise and anticlockwise direction when seen from top. Then, to determine the active/reactive gyroscopic couple vector, the following procedure is used.

- Turn the spin vector through 900 in the direction of precession on the XOZ plane
- The turned spin vector will then correspond to the direction of active gyroscopic couple/torque vector
- The reactive gyroscopic couple/torque vector is taken opposite to active gyro vector direction

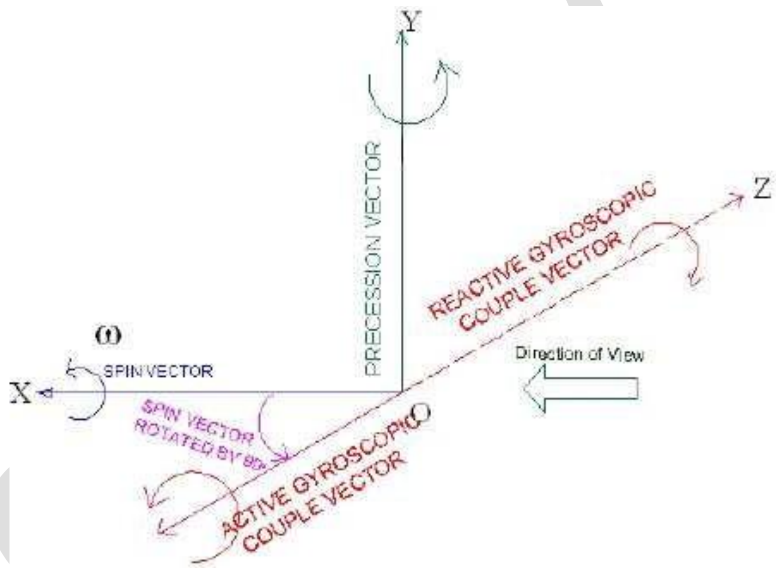


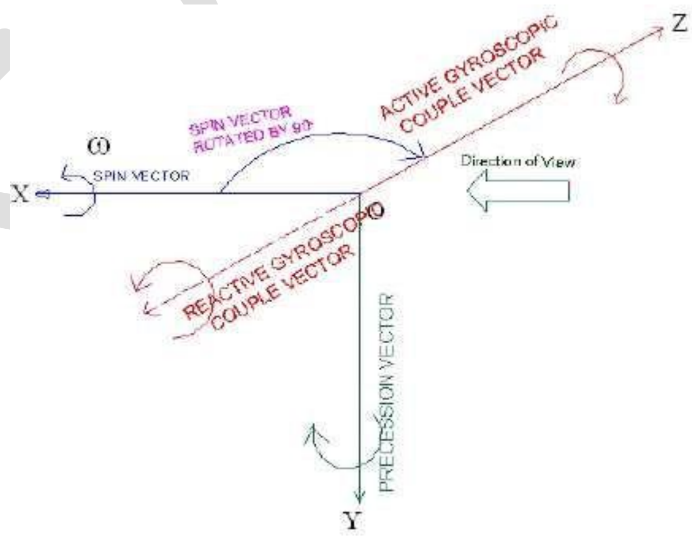


Case (ii):

Consider a rotor rotating in clockwise direction when seen from the right (Fig.7 and Fig. 8), and to precess the spin axis about precession axis in clockwise and anticlockwise direction when seen from top. Then, to determine the active/reactive gyroscopic couple vector,

- Turn the spin vector through 900 in the direction of precession on the XOZ plane
- The turned spin vector will then correspond to the direction of active gyroscopic couple/torque vector
- The reactive gyroscopic couple/torque vector is taken opposite to active gyro vector direction





The resisting couple/ reactive couple will act in the direction opposite to that of the gyroscopic couple. This means that, whenever the axis of spin changes its direction, a **gyroscopic couple** is applied to it through the bearing which supports the spinning axis.

GYROSCOPIC EFFECT ON SHIP

Gyroscope is used for stabilization and directional control of a ship sailing in the rough sea. A ship, while navigating in the rough sea, may experience the following three different types of motion:

(i) Steering—The turning of ship in a curve while moving forward

(ii) Pitching—The movement of the ship up and down from horizontal position in a vertical plane about transverse axis

(iii)Rolling—Sideway motion of the ship about longitudinal axis

For stabilization of a ship against any of the above motion, the major requirement is that the gyroscope shall be made to precess in such a way that reaction couple exerted by the rotor opposes the disturbing couple which may act on the frame.

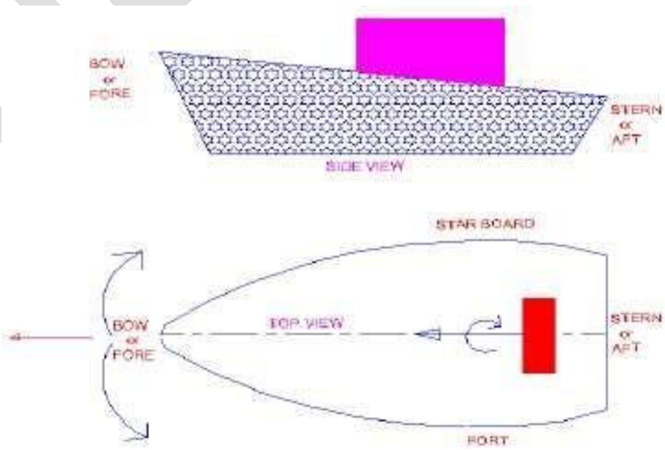
Ship Terminology

(i) Bow – It is the fore end of ship

(ii) Stern – It is the rear end of ship

(iii) Starboard – It is the right hand side of the ship looking in the direction of motion

(iv) Port – It is the left hand side of the ship looking in the direction of motion

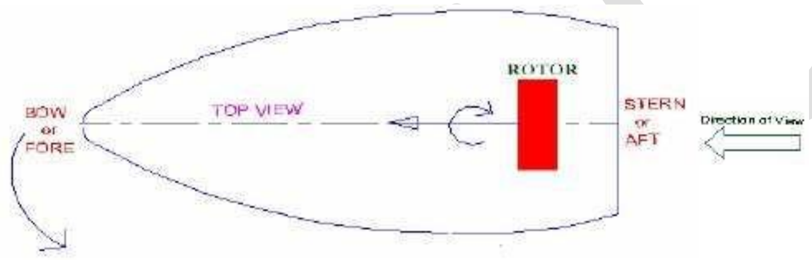


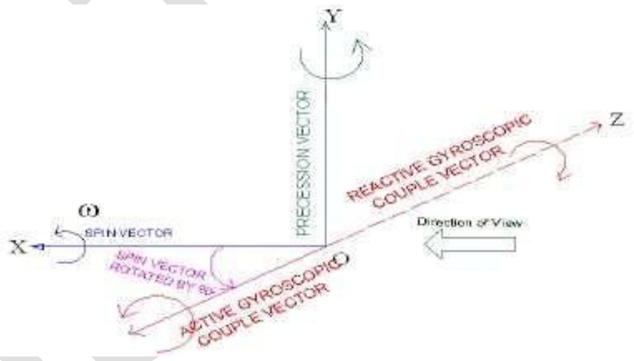
Consider a gyro-rotor mounted on the ship along longitudinal axis (X-axis) as shown in Fig.10 and rotate in clockwise direction when viewed from rear end of the ship. The angular speed of the rotor is □ rad/s. The direction of angular momentum vector *oa*, based on direction of rotation of rotor, is decided using right hand thumb rule as discussed earlier. The gyroscopic effect during the three types of motion of ship is discussed.

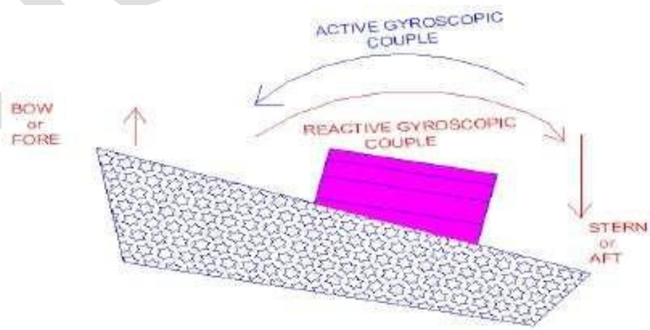
Gyroscopic effect on Steering of ship

(i) Left turn with clockwise rotor

When ship takes a left turn and the **rotor rotates in clockwise direction** viewed from stern, the gyroscopic couple act on the ship is analyzed in the following way.



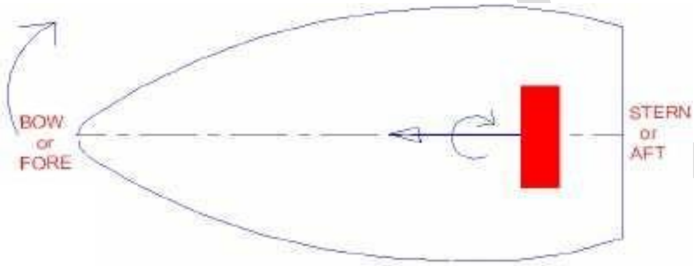


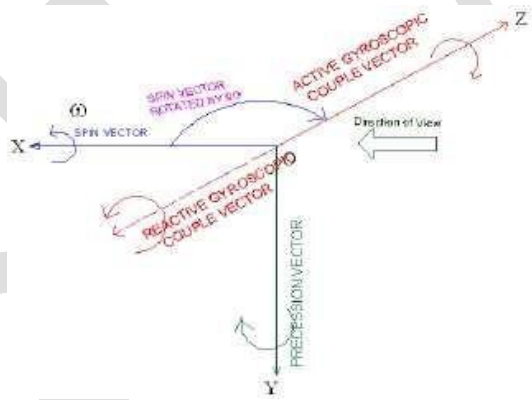


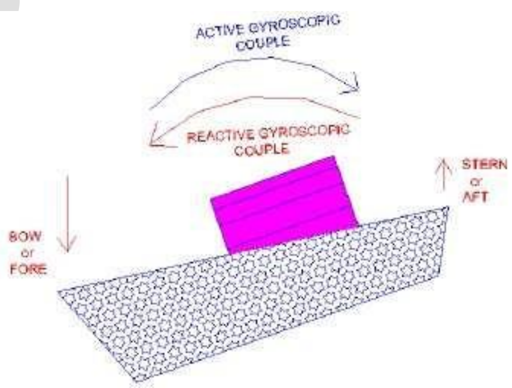
Note that, always reactive gyroscopic couple is considered for analysis. From the above analysis (Fig.), the couple acts over the ship between stern and bow. This reaction couple tends to raise the front end (bow) and lower the rear end (stern) of the ship.

(ii) Right turn with clockwise rotor

When ship takes a right turn and the **rotor rotates in clockwise direction** viewed from stern, the gyroscopic couple acts on the ship is analyzed (Fig 14). Again, the couple acts in vertical plane, means between stern and bow. Now the reaction couple tends to lower the bow of the ship and raise the stern.

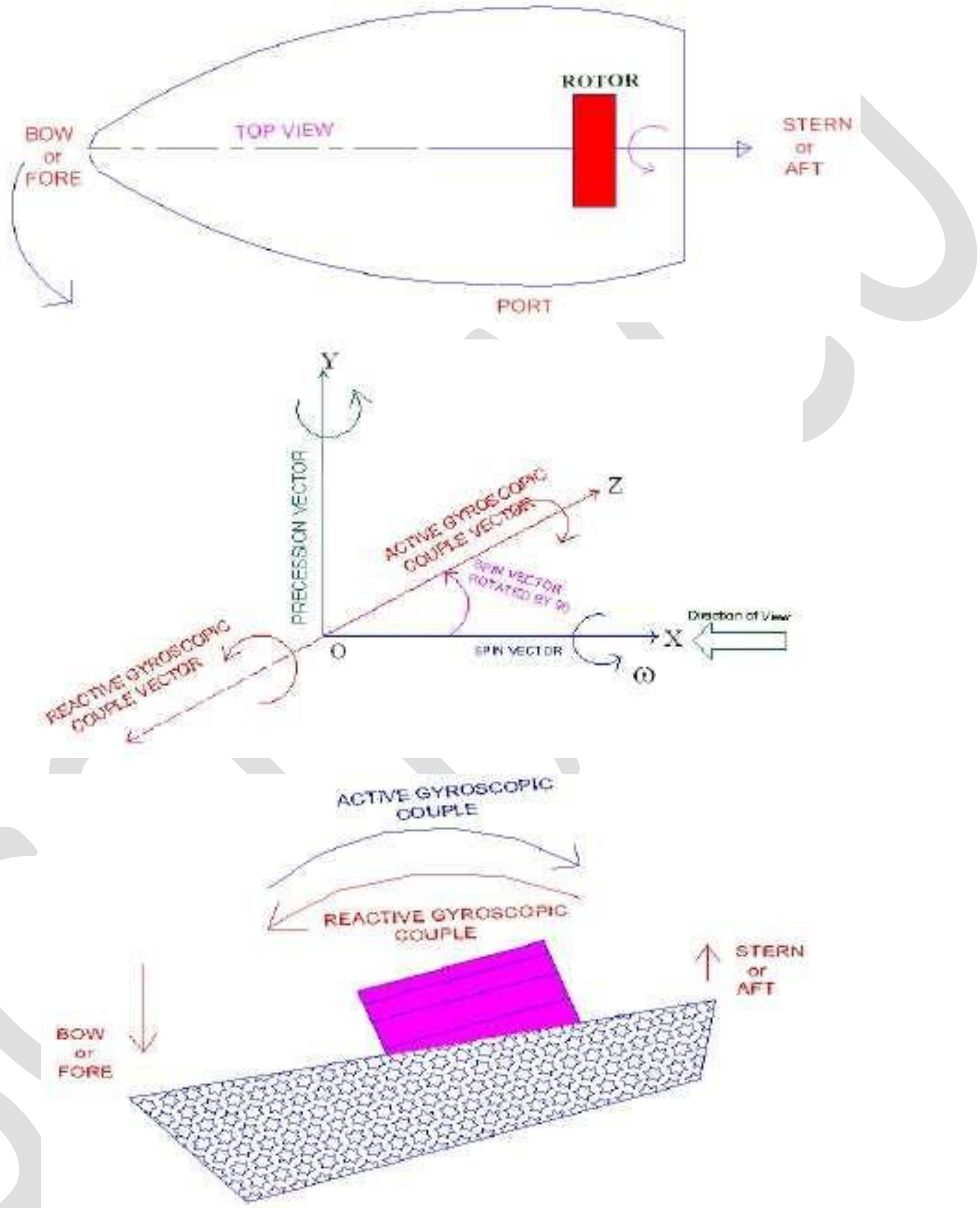






(iii) Left turn with anticlockwise rotor

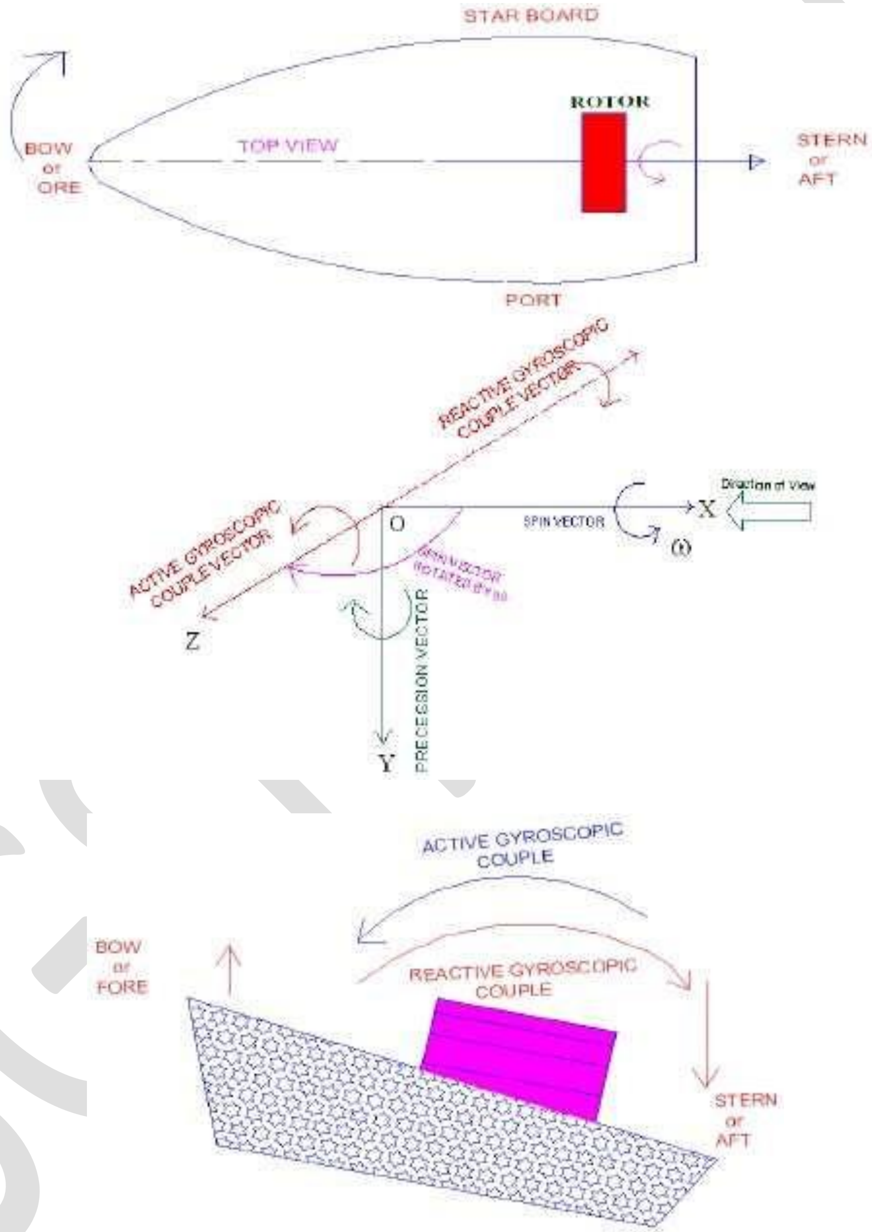
When ship takes a left turn and the **rotor rotates in anticlockwise direction** viewed from stern, the gyroscopic couple act on the ship is analyzed in the following way (Fig.).



The couple acts over the ship is between stern and bow. This reaction couple tends to press or dip the front end (bow) and raise the rear end (stern) of the ship.

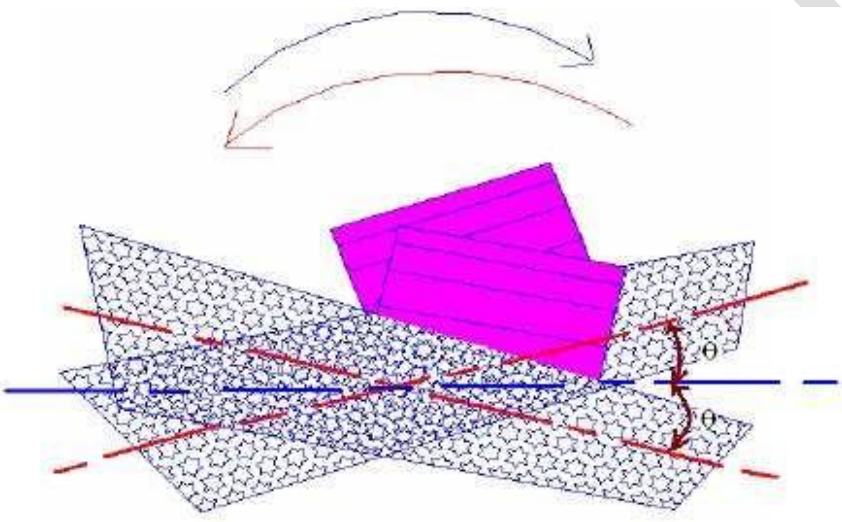
(iv) Right turn with anticlockwise rotor

When ship takes a right turn and the **rotor rotates in anticlockwise direction** viewed from stern, the gyroscopic couple act on the ship is according to Fig 20. Now, the reaction couple tends to raise the bow of the ship and dip the stern



Gyroscopic effect on Pitching of ship

The pitching motion of a ship generally occurs due to waves which can be approximated as sine wave. During pitching, the ship moves up and down from the horizontal position in vertical plane (Fig.)



Let θ = angular displacement of spin axis from its mean equilibrium position

A = amplitude of swing

(= angle in degree $\times \frac{2\pi}{360°}$)

and ω_0 = angular velocity of simple hormonic motion $\left(= \frac{2\pi}{\text{time period}}\right)$

The angular motion of the rotor is given as

$$\theta = A \sin \omega_0 t$$

Angular velocity of precess:

$$\omega_p = \frac{d\theta}{dt}$$

$$= \frac{d}{dt}(A \sin \omega_0 t)$$

or

$$\omega_p = A\omega_0 \cos \omega_0 t$$

The angular velocity of precess will be maximum when $\cos \omega_0 t = 1$

or

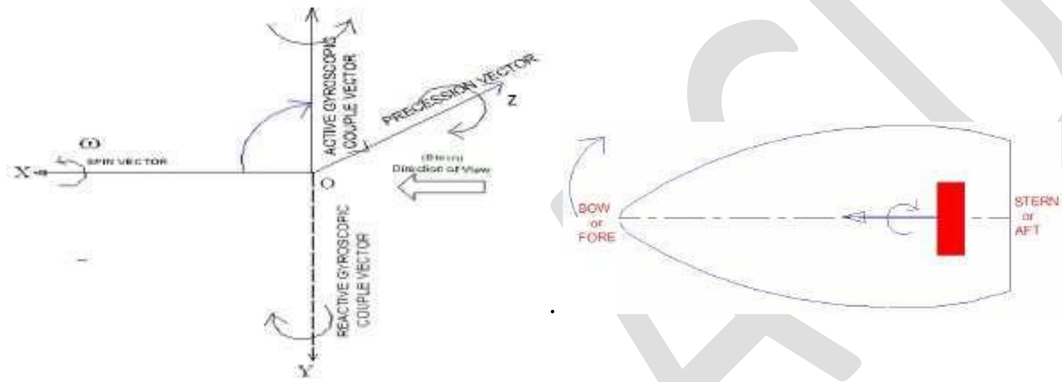
$$\omega_{p\max} = A\omega_0$$

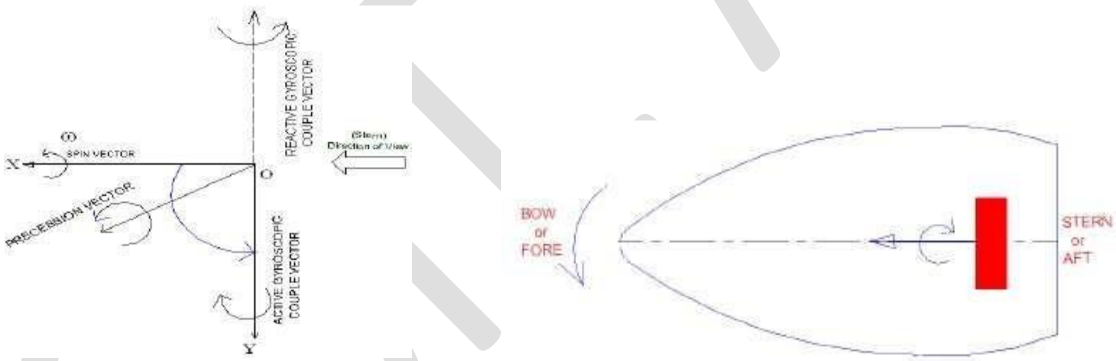
$$= A \times \frac{2\pi}{t}$$

Thus the gyroscopic couple:

$$C = I\,\omega\,\omega_p$$

Consider a rotor mounted along the longitudinal axis and rotates in clockwise direction when seen from the rear end of the ship. The direction of momentum for this condition is shown by vector *ox* (Fig.24). When the ship moves up the horizontal position in vertical plane by an angle □□ from the axis of spin, the rotor axis (X-axis) processes about Z- axis in XY-plane and for this case Z-axis becomes precession axis. The gyroscopic couple acts in anticlockwise direction about Y-axis and the reaction couple acts in opposite direction, i.e. in clockwise direction, which tends to move towards **right side** (Fig.25). However, when the ship pitches down the axis of spin, the direction of reaction couple is reversed and the ship turns towards **left side** (Fig.)





Similarly, for the anticlockwise direction of the rotor viewed from the rear end (Stern) of the ship, the analysis may be done.

Gyroscopic effect on Rolling of ship.

The axis of the rotor of a ship is mounted along the longitudinal axis of ship and therefore, there is **no** precession of this axis. Thus, no effect of gyroscopic couple on the ship frame is formed when the ship rolls

Gyroscopic Effect on Aeroplane

Aeroplanes are subjected to gyroscopic effect when it taking off, landing and negotiating left or right turn in the air.

Let

ω = Angular velocity of the engine rotating parts in rad/s, m
= Mass of the engine and propeller in kg,

rW = Radius of gyration in m,

I = Mass moment of inertia of engine and propeller in kg m2, V
= Linear velocity of the aeroplane in m/s,

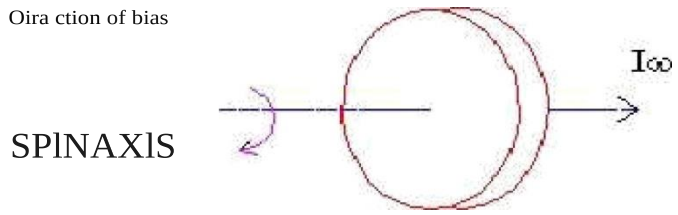
R = Radius of curvature in m,

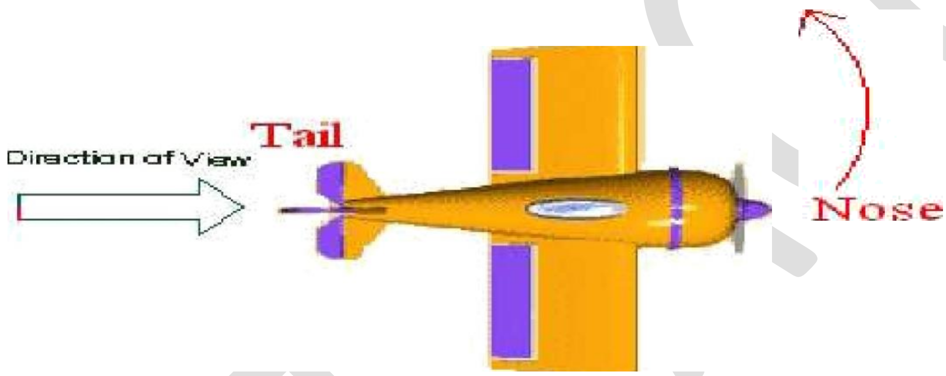
ωp =Angular velocity of precession =v/R rad/s

Gyroscopic couple acting on the aero plane **= C = I ω ωp**

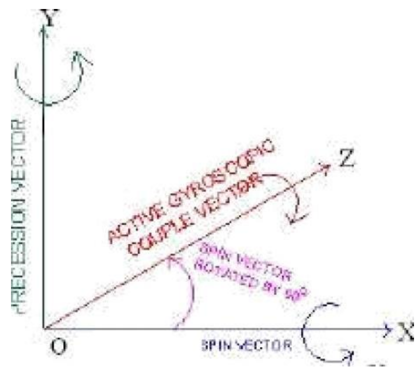
Let us analyze the effect of gyroscopic couple acting on the body of the aero plane for various conditions.

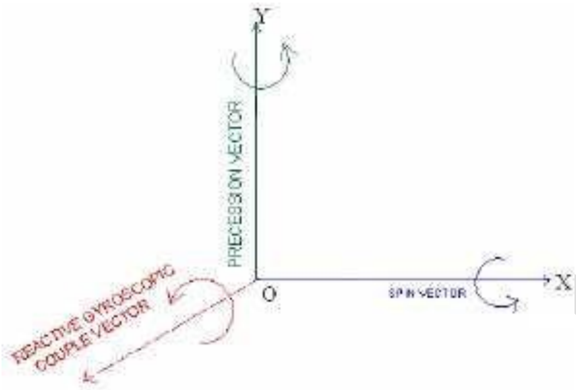
Case (i): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane turns towards LEFT


Oira ction of bias
SPlNAXlS
Iω

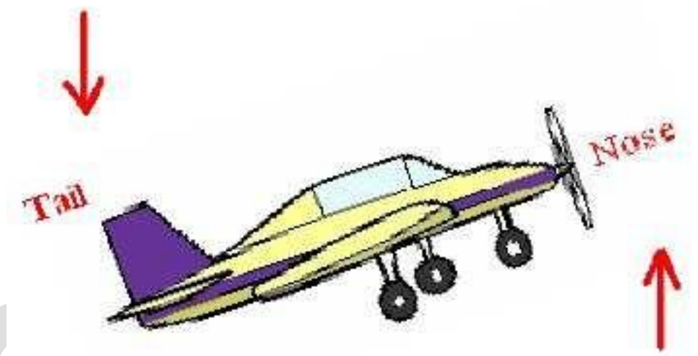

Direction of View
Tail
Nose


Direction of View
O
SPIHSECTOR
X
Iω

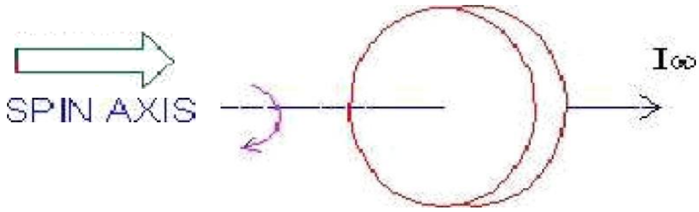

Y
Z
X
O
PRECESSION VECTOR
ACTIVE GYROSCOPIC COUPLE VECTOR
SPIN VECTOR ROTATED BY 90°
SPIN VECTOR

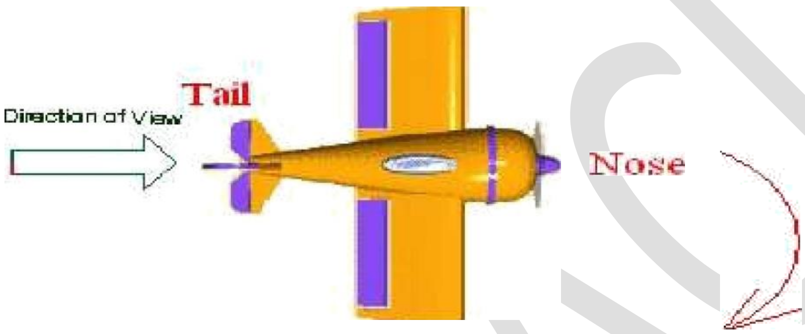


According to the analysis, the reactive gyroscopic couple tends to dip the tail and raise the nose of aeroplane.

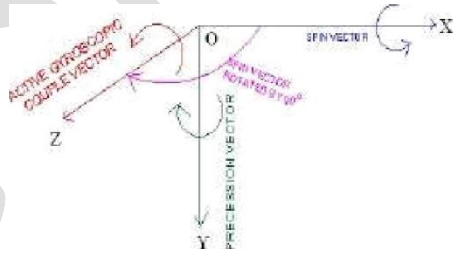


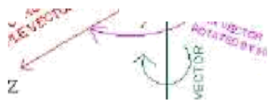
Case (ii): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane turns towards RIGHT


SPIN AXIS
Iω

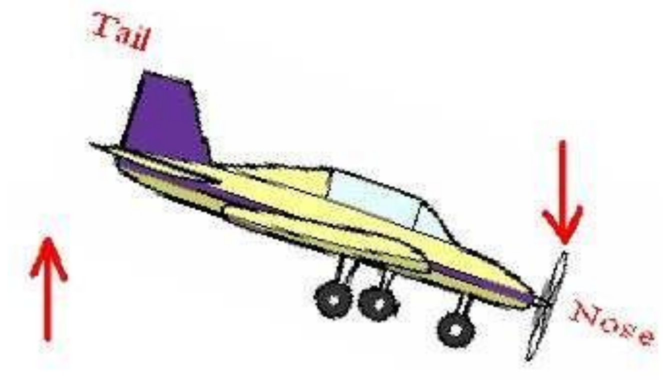

Direction of View
Tail
Nose


Direction of View
O
SPIfi VECTOR
X
Iω

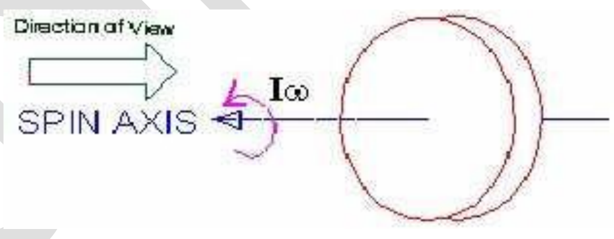

O
SPIN VECTOR
X
ACTIVE GYROSCOPIC COUPLE VECTOR
Z
PRECESSION VECTOR
Y

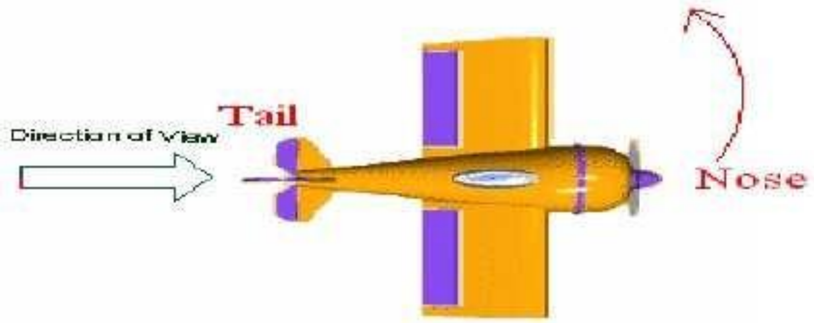

Z
VECTOR

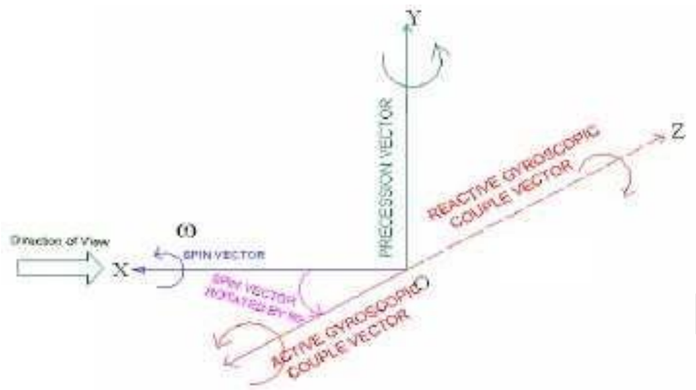
According to the analysis, the reactive gyroscopic couple tends to raise the tail and dip the nose of aeroplane.



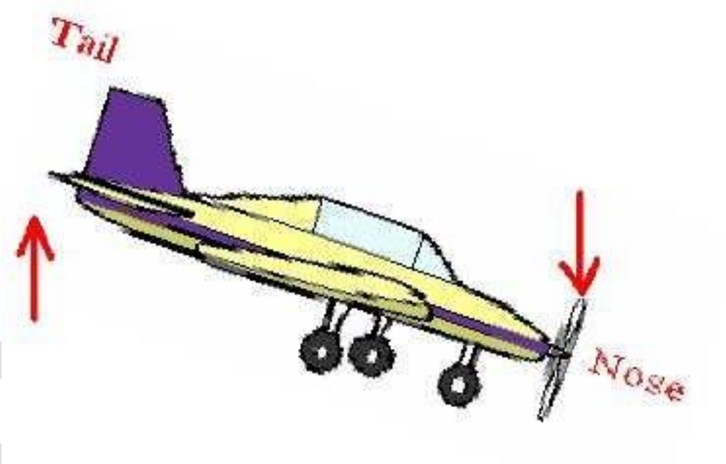
Case (iii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane turns towards LEFT



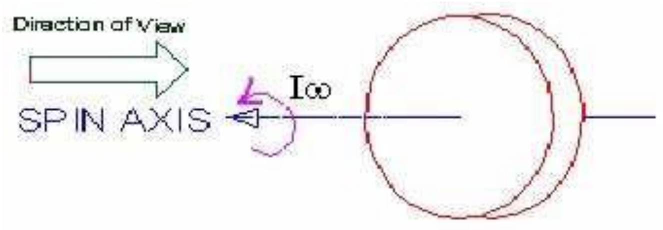


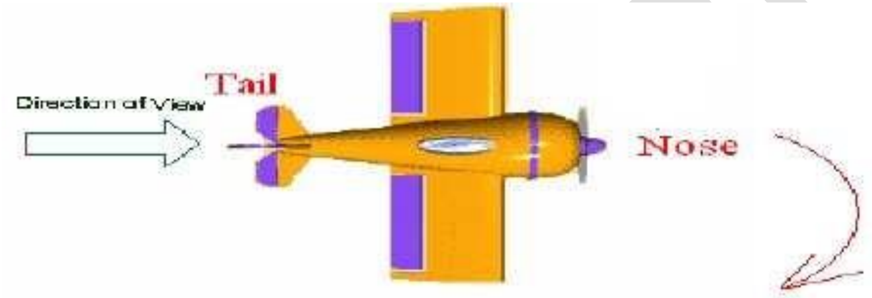


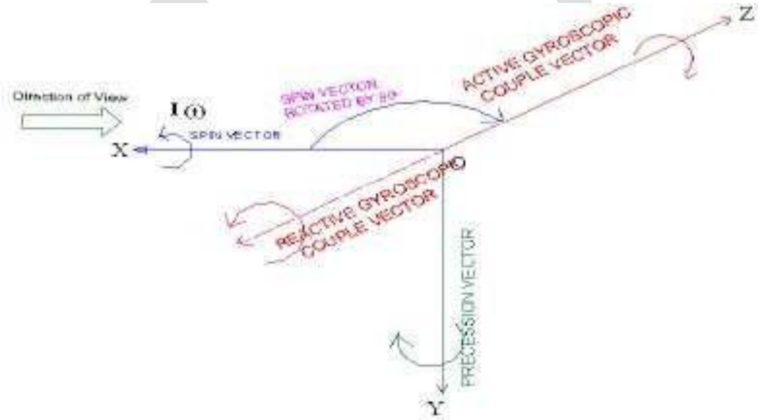
The analysis indicates, the reactive gyroscopic couple tends to raise the tail and dip the nose of aeroplane.



Case (iv): **PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane turns towards RIGHT**

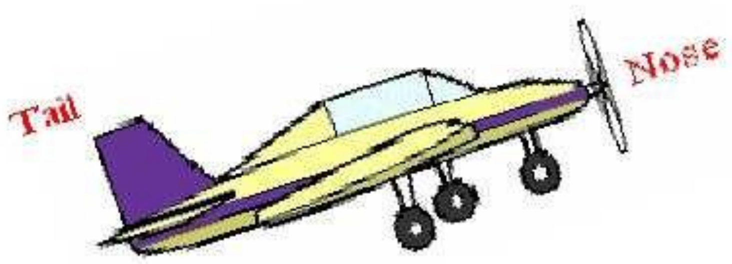


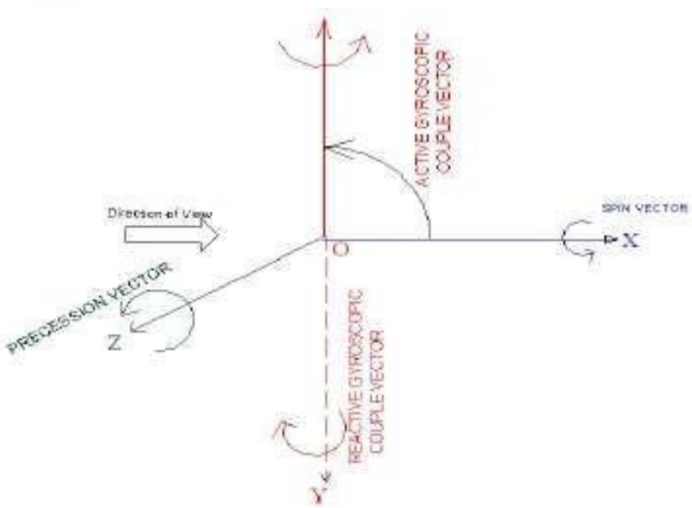




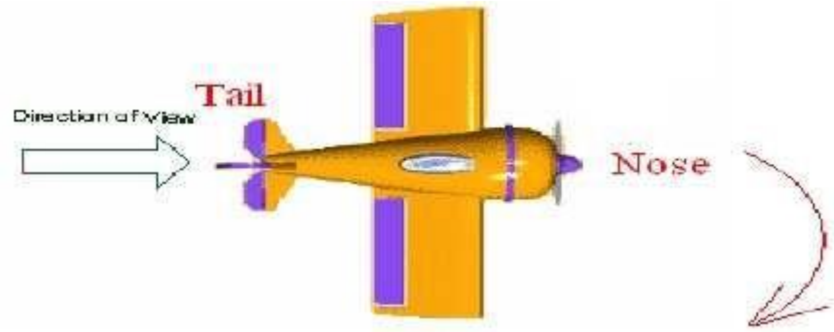
The analysis shows, the reactive gyroscopic couple tends to raise the tail and dip the nose of aeroplane.

Case (v): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane takes off or nose move upwards

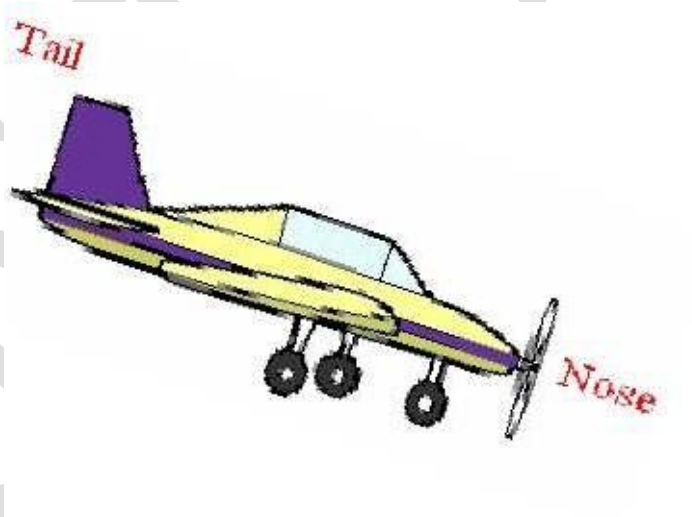


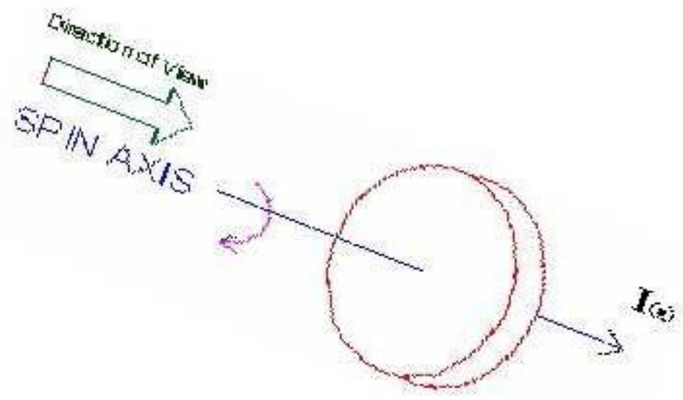


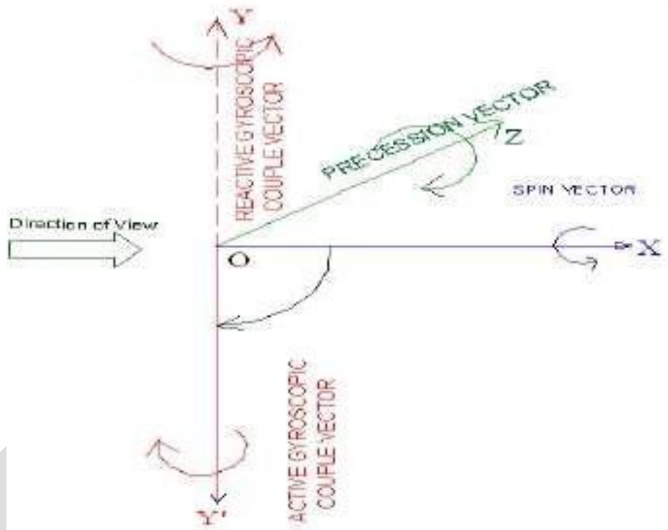
The analysis show, the reactive gyroscopic couple tends to turn the nose of aeroplane toward right



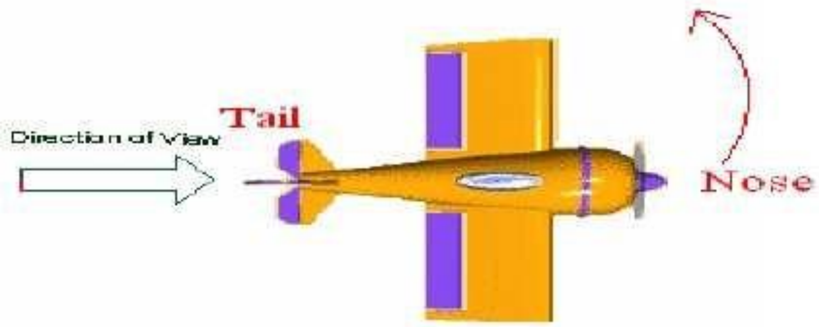
Case (vi): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane is landing or nose move downwards



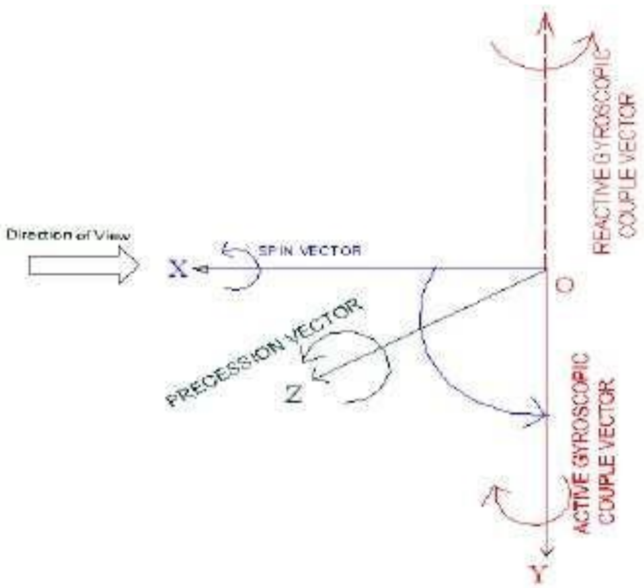




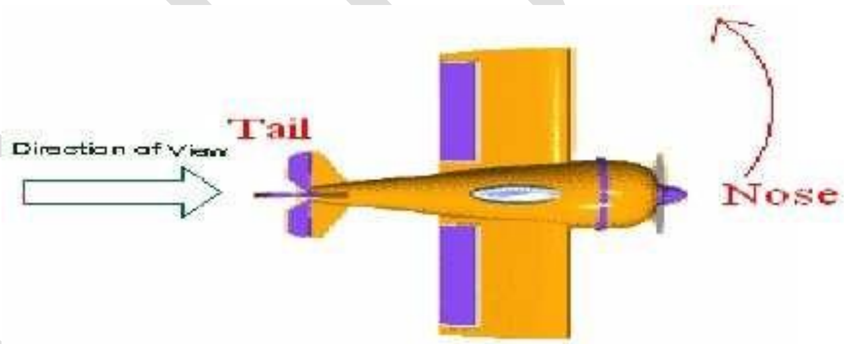
The reactive gyroscopic couple tends to turn the nose of aeroplane toward left



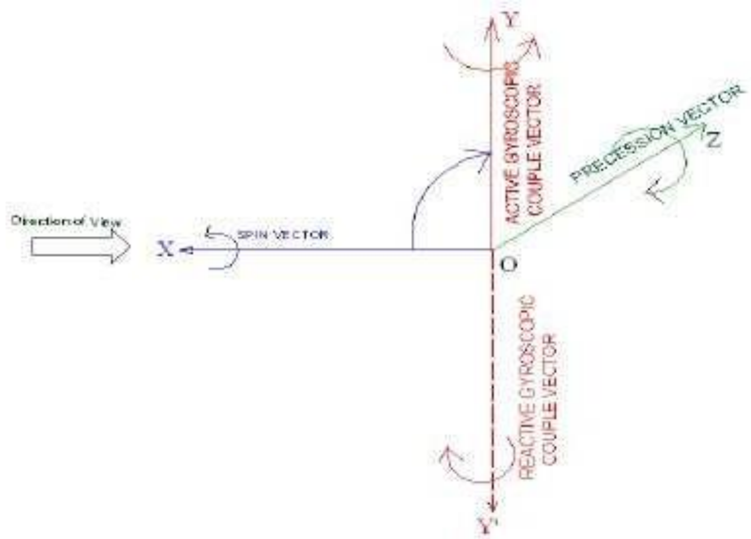
Case (vii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane takes off or nose move upwards



The reactive gyroscopic couple tends to turn the nose of aeroplane toward left



Case (viii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane is landing or nose move downwards



The analysis show, the reactive gyroscopic couple tends to turn the nose of aeroplane toward right

Stability of Automotive Vehicle

A vehicle running on the road is said to be stable when no wheel is supposed to leave the road surface. In other words, the resultant reactions by the road surface on wheels should act in upward direction. For a moving vehicle, one of the reaction is due to gyroscopic couple produced by the rotating wheels and rotating parts of the engine. Let us discuss stability of two and four wheeled vehicles when negotiating a curve/turn.

Stability of Two Wheeler negotiating a turn

Fig shows a two wheeler vehicle taking **left turn** over a curved path. The vehicle is inclined to the vertical for equilibrium by an angle known as angle of heel.

Let

m = Mass of the vehicle and its rider in kg,

W = Weight of the vehicle and its rider in newtons = *m.g*, *h*
= *Height of the centre of* gravity of the vehicle and rider, *rW*
= *Radius of the wheels,*

R = Radius of track or curvature,

IW = Mass moment of inertia of each wheel,

IE = Mass moment of inertia of the rotating parts of the engine,

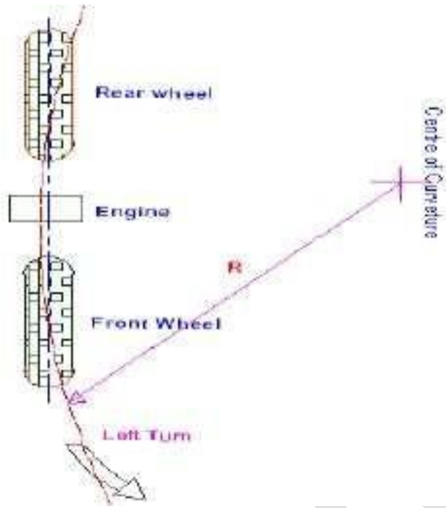
ωW = Angular velocity of the wheels,

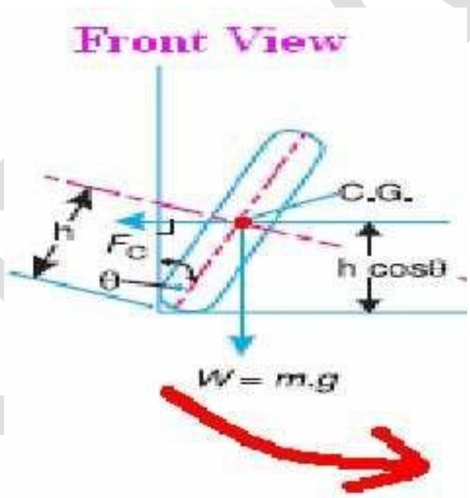
ωE = Angular velocity of the engine rotating parts,

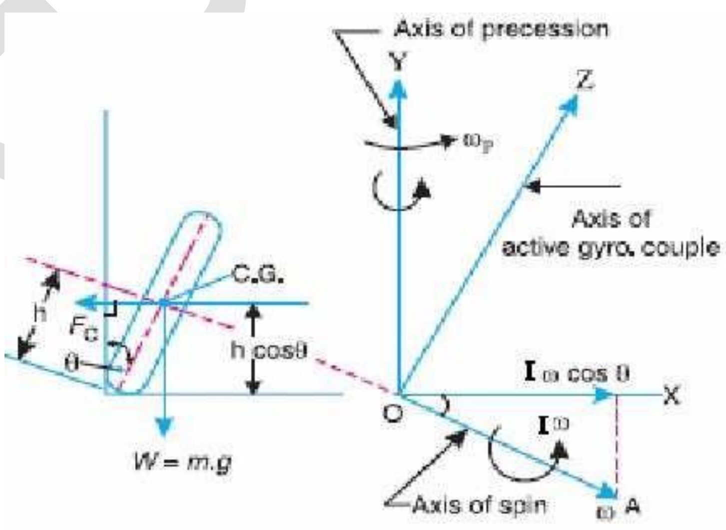
G = Gear ratio = ωE

v = Linear velocity of the vehicle = $\omega_w \times r_w$,

θ = Angle of heel. It is inclination of the vehicle to the vertical for equilibrium







Let us consider the effect of the gyroscopic couple and centrifugal couple on the wheels.

1. Effect of Gyroscopic Couple

We know that, $V = \omega_W \times r_W$

$\omega_E = G \cdot \omega_W$ or

Angular momentum due to wheels = 2 $I_w \omega_W$

Angular momentum due to engine and transmission = $I_E \omega_E$

Total angular momentum ($I \times \omega$) = 2 $I_w \omega_W \pm I_E \omega_E$

$$= 2I_w \frac{v}{r_w} \pm I_E G \frac{v}{r_w}$$

$$= \frac{v}{r_w}(2I_w \pm GI_E)$$

Velocity of precession = ω_p

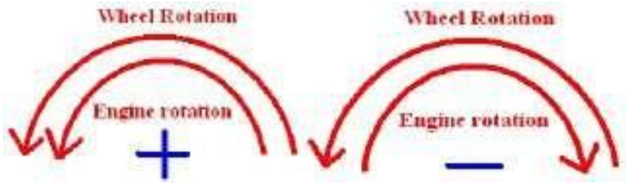
It is observed that, when the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane as shown in Fig… This angle is known as 'angle of heel'. In other words, the axis of spin is inclined to the horizontal at an angle θ , as shown in Fig.73 Thus, the angular momentum vector I ω due to spin is represented by OA inclined to OX at an angle θ. But, the precession axis is in vertical. Therefore, the spin vector is resolved along OX.

Gyroscopic Couple,

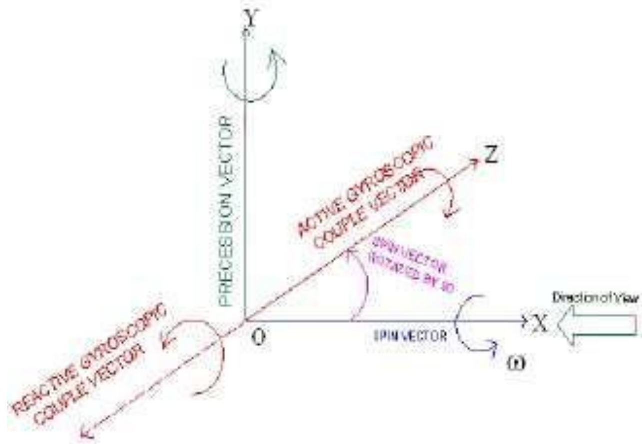
$$C_g = (I\omega)\cos\theta \times \omega_p$$

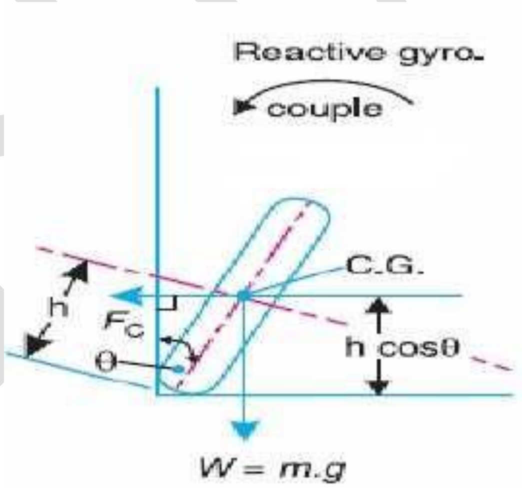
$$C_g = \frac{v^2}{Rr_w}(2I_w \pm GI_E)\cos\theta$$

Note: When the engine is rotating in the same direction as that of wheels, then the positive sign is used in the above equation. However, if the engine rotates in opposite direction to wheels, then negative sign is used.

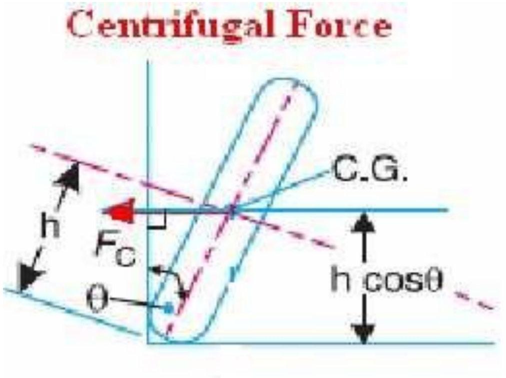


The gyroscopic couple will act over the vehicle outwards i.e., in the anticlockwise direction when seen from the front of the two wheeler. This couple tends to overturn/topple the vehicle in the outward direction as shown in Fig…





2. Effect of Centrifugal Couple

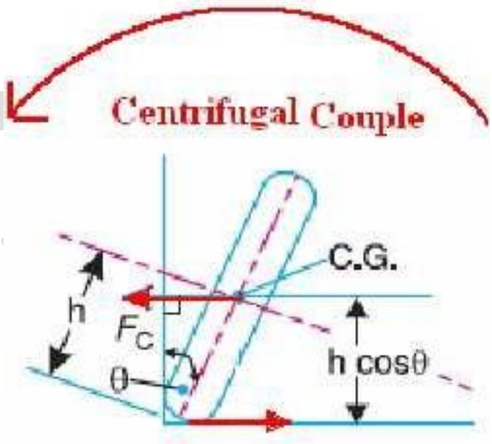


Centrifugal force,

$$F_c = \frac{mv^2}{R}$$

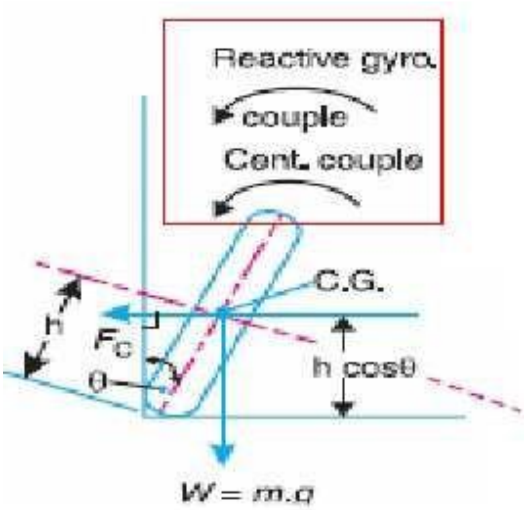
Centrifugal Couple

$$C_c = F_c \times h\cos\theta$$
$$= \frac{mv^2}{R} h\cos\theta$$



The Centrifugal couple will act over the two wheeler outwards i.e., in the anticlockwise direction when seen from the front of the two wheeler. This couple tends to overturn/topple the vehicle in the outward direction as shown in Fig.

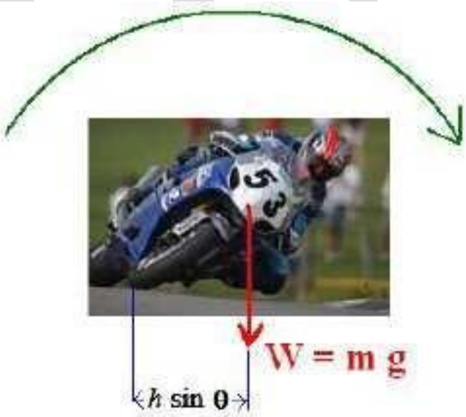
Therefore, the total Over turning couple: C = Cg + Cc



$$C = \frac{v^2}{Rr}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R}h\cos\theta$$

For the vehicle to be in equilibrium, overturning couple should be equal to balancing couple acting in clockwise direction due to the weight of the vehicle and rider.

C = mgh sin

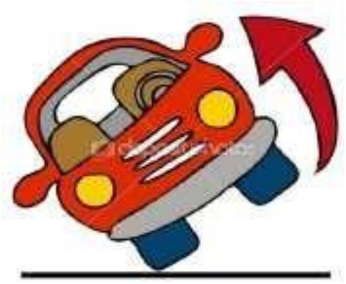


For the stability, overturning couple must be equal to balancing couple,

$$\frac{v^2}{Rr_w}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R}h\cos\theta = mgh\sin\theta$$

Therefore, from the above equation, the value of angle of heel (θ) may be determined, so that the vehicle does not skid. Also, for the given value of θ, the maximum vehicle speed in the turn with out skid may be determined.

Stability of Four Wheeled Vehicle negotiating a turn.



Consider a four wheels automotive vehicle as shown in Figure 82. The engine is mounted at the rear with its crank shaft parallel to the rear axle. The centre of gravity of the vehicle lies vertically above the ground where total weight of the vehicle is assumed to be acted upon.

Let

m = Mass of the vehicle (kg)

W = Weight of the vehicle (N) = *m.g,*

h = Height of the centre of gravity of the vehicle (m)

rW = Radius of the wheels (m)

R = Radius of track or curvature (m)

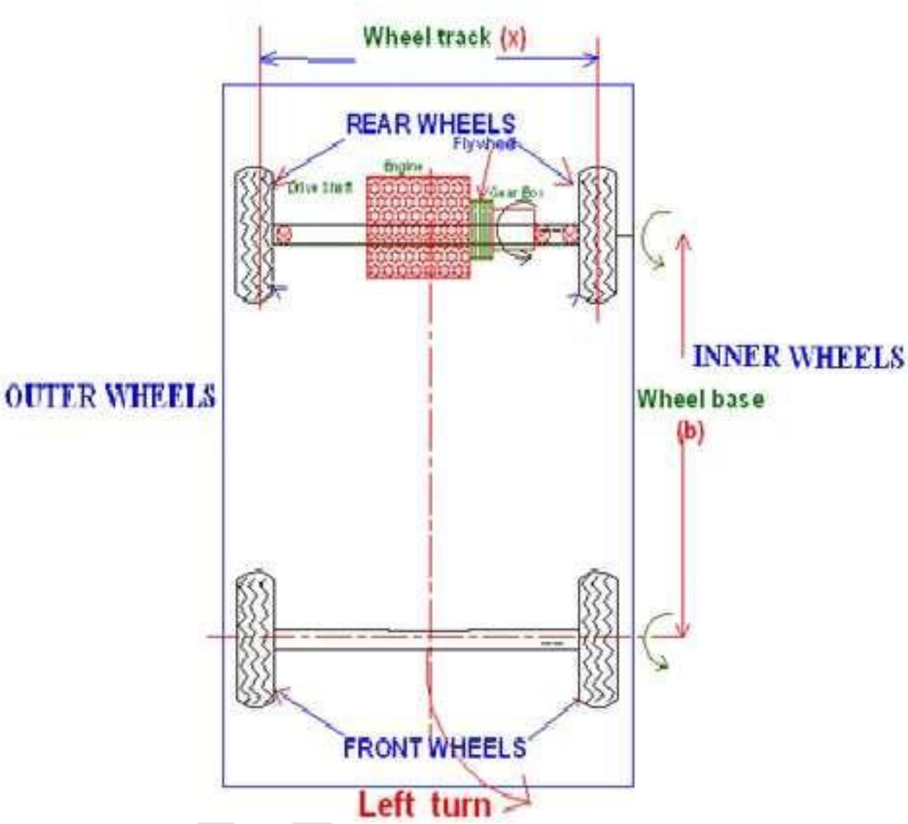
IW = Mass moment of inertia of each wheel (kg-m2)

IE = Mass moment of inertia of the rotating parts of the engine (kg-m2)

ωW = Angular velocity of the wheels (rad/s)
ωE = Angular velocity of the engine (rad/s) *G*
= Gear ratio = ωE / ωW,

v = Linear velocity of the vehicle (m/s)= ωW × rW,

x = Wheel track (m) b
= Wheel base (m)



(i) Reaction due to weight of Vehicle

Weight of the vehicle. Assuming that weight of the vehicle *(mg)* is equally distributed over four wheels. Therefore, the force on each wheel acting downward is m*g/4* and the reaction by the road surface on the wheel acts in upward direction.

$$R_w = \frac{mg}{4}$$

(ii) Effect of Gyroscopic couple due to Wheel

Gyroscopic couple due to four wheels is,

$$C_W = 4 I_W \omega \omega_p$$

(iii) Effect of Gyroscopic Couple due to Engine

Gyroscopic couple due to rotating parts of the engine

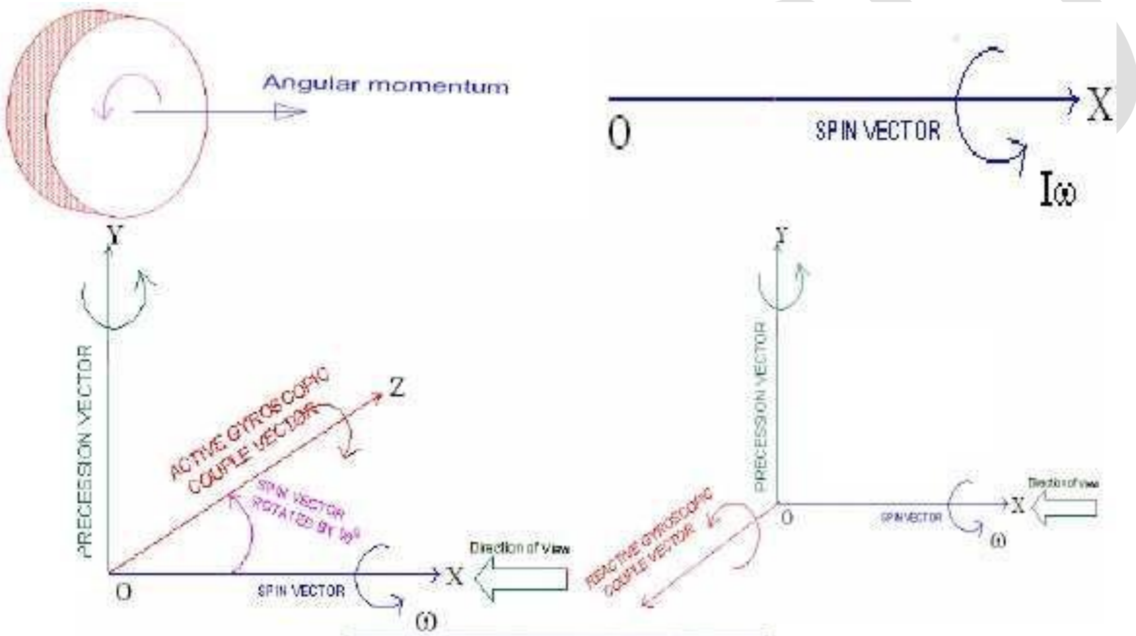
$$C_E = I_E \omega \omega_p = I_E G \omega \omega_p$$

Therefore, Total gyroscopic couple:

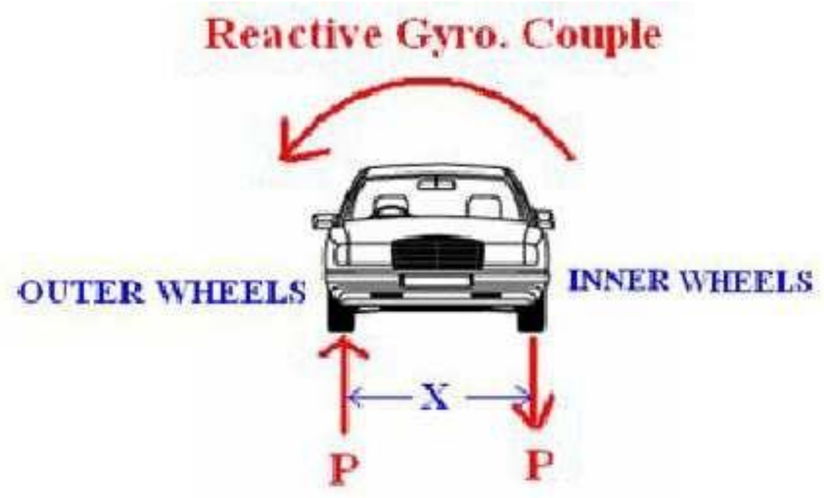
$$C_g = C_W + C_E = \omega\,\omega_p\,(4I_W \pm I_E G)$$

When the wheels and rotating parts of the engine rotate in the same direction, then positive sign is used in the above equation. Otherwise negative sign should be considered.

Assuming that the vehicle takes a left turn, the reaction gyroscopic couple on the vehicle acts between outer and inner wheels.



This gyroscopic couple tends to press the outer wheels and lift the inner wheels



Due to the reactive gyroscopic couple, vertical reactions on the road surface will be produced. The reaction will be vertically upwords on the outer wheels and vertically downwords on the inner wheels. Let the magnitude of this reaction at the two outer and inner wheels be P Newtons, then,

$$\mathbf{P} \times X = C_g$$

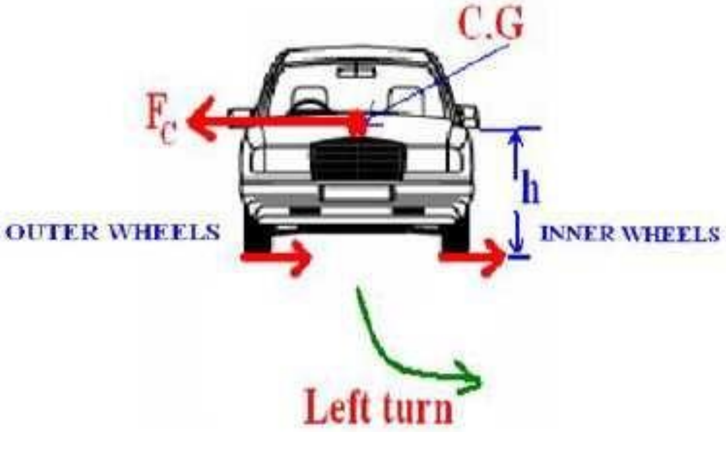
$$\mathbf{P} = \frac{C_g}{x}$$

Road reaction on each outer/Inner wheel,

$$\frac{P}{2} = \frac{C_g}{2X}$$

(iii)**Effect of Centrifugal Couple**

When a vehicle moves on a curved path, a centrifugal force acts on the vehicle in outward direction through the centre of gravity of the vehicle(Fig…)



Centrifugal force,

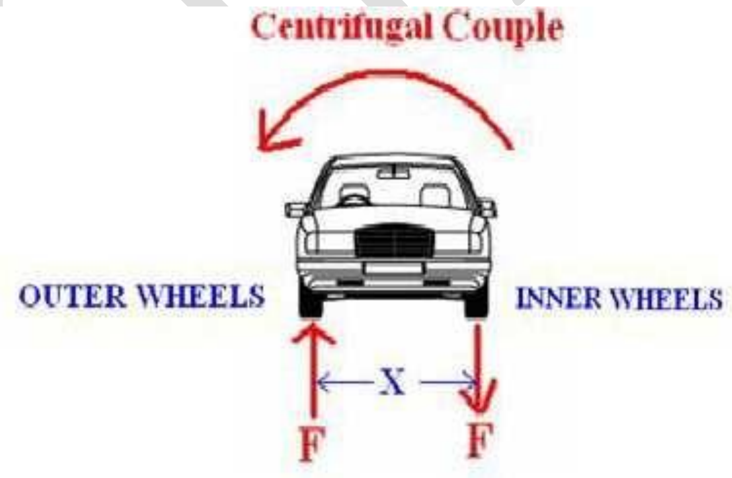
$$F_c = m\omega_p^2 R = \frac{mv^2}{R}$$

This force forms a Centrifugal couple.

$$C_c = \frac{mv^2 h}{R}$$

This centrifugal couple tends to press the outer and lift the inner

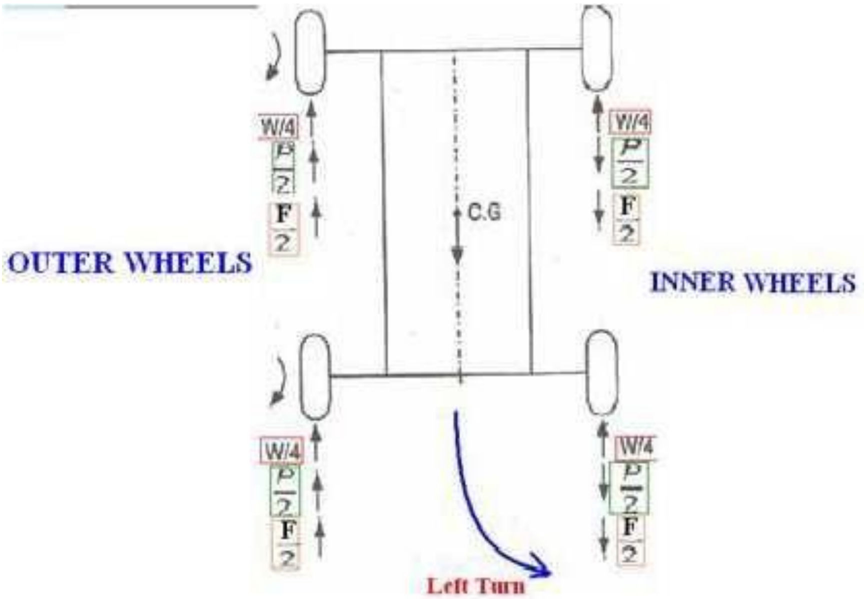
Due to the centrifugal couple, vertical reactions on the road surface will be produced. The reaction will be vertically upwords on the outer wheels and vertically downwords on the inner wheels. Let the magnitude of this reaction at the two outer and inner wheels be F Newtons, then,



Road reaction on each outer/Inner wheel,

$$\frac{F}{2} = \frac{C_c}{2X}$$

The reactions on the outer/inner wheels are as follows,



Total vertical reaction at each outer wheels

$$P_o = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

Total vertical reaction at each inner wheels

$$P_i = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$